Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics: Section 1
Test 3

Name:_____

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
A tree T has at least two vertices and maximum degree 3. Show that the number n_1 of vertices of degree 1 and the number n_3 of vertices of degree 3 satisfy the relation

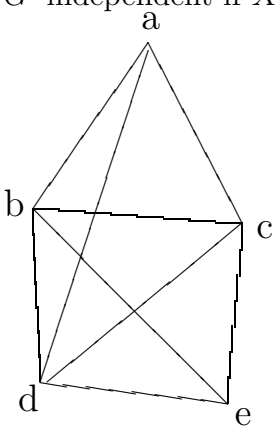
$$n_1 = n_3 + 2.$$

[Hint: find 2 equations satisfied by n_1, n_2, n_3 or use induction.]

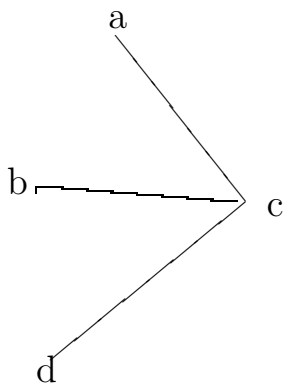
Q2: (33pts)
Let $G = (V, E)$ be a connected graph. A Red-Blue colouring of the edges E is *balanced* if each vertex is incident with the same number of Red edges as Blue edges. Show that G has a balanced colouring iff it is Eulerian.

There is an error in this question. Give an Eulerian graph without any balanced colourings. The statement is true if we consider bipartite graphs.

Q3: (34pts)
Prove that a graph $G = (V, E)$ is bipartite iff every subgraph $G' = (V', E')$ of G has a G'-independent set of size at least $|V'|/2$. A set $X \subseteq V'$ is G'-independent if X contains no edges of G'.



G



H

{a,b,d} is H-independent

[Hint: think about the relationship between bipartite graphs and odd cycles.]

Continue your answer to Q3.