Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics
Test 3

Name:____

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
Solve the following linear recurrence:

$$u_n - 6u_{n-1} + 9u_{n-2} = n(n-1)3^n, \qquad n \geq 2.$$

$u_0 = u_1 = 0.$
Be sure to show your working.

Q2: (34pts)
Let B_n be the number of partitions of $\{1, 2, \dots, n\}$ – the Bell numbers. Use inclusion-exclusion to show that the number of partitions of $\{1, 2, \dots, n\}$ which have no singleton sets is

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} B_{n-k}.$$

Q3: (33pts)

(a) Let u_k denote the number of ways of partitioning $\{1, 2, \ldots, 2k\}$ into k sets of size 2. Find an expression for u_k.

(b) Use inclusion-exclusion to show that the number of permutations of $\{1, 2, \ldots, n\}$ which have no cycles of length 2 is

$$\sum_{k=0}^{n/2} (-1)^k \binom{n}{2k} u_k (n-2k)!.$$

(Hint: For a two element set $e = \{x, y\} \subseteq [n]$ let A_e be the set of permutations π such that $\pi(x) = y$ and $\pi(y) = x$.)