Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics
Test 2

Name:________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
m indistinguishable balls are randomly colored using n colors.

1. What is the expected number of colors that are used **exactly** twice.
2. What is the expected number of colors that are used **at least** twice.

Q2: (33pts)
Solve the following linear recurrence:

$$u_n - 6u_{n-1} + 8u_{n-2} = 0, \qquad\qquad n \geq 2.$$

$u_0 = 4$, $u_1 = 10$.
Be sure to show your working.

Q3: (34pts)
Let A_n denote the set of strings of length n over $\{0, 1\}$. Let B_n be those strings in A_n which do not contain 111 as a substring and let $b_n = |B_n|$.

- What are b_0, b_1, b_2, b_3?
- Prove that for $n \geq 4$,

$$b_n = b_{n-1} + b_{n-2} + b_{n-3}.$$