Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics
Test 1

Name:______________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
Let $a_{n,m}$ denote the number of ways of colouring m indistinguishable balls with n colours such that colour i is used at least i times for $i = 1, 2, \dots, n$. Show that

$$a_{n,m} = \binom{m - \frac{n^2-n+2}{2}}{n-1}.$$

Q2: (33pts)
Let $a_1, a_2, \dots, a_n$ be an arbitrary collection of integers.
Let $S_0 = 0, S_1 = a_1, S_2 = a_1 + a_2, \dots, S_n = a_1 + a_2 + \cdots a_n$.

(i) Show that there exist $i < j$ such that $S_i \equiv S_j \pmod{n}$.

(ii) Using (i) show that there exist $k \leq l$ such that n divides $a_k + \cdots + a_l$.

Q3: (34pts)
A woman starts at (0,0) of an $n \times n$ grid i.e. the top right hand corner is (n, n). She takes a monotone walk to (n, n) but when she has a choice of 2 moves, she always chooses randomly. Show that the probability she never drops below the diagonal is $\frac{1}{n+1}$.
(Hint: She is choosing a random path uniformly from $PATHS(n, n)$.)