Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics
Test 1

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
Show that

$$\sum_{k=0}^{n} \frac{1}{k+1}\binom{n}{k} = \frac{1}{n+1}(2^{n+1}-1).$$

Q2: (33pts)
Show that the number of strings of p 0's and q 1's where each pair of 1's is separated by at least 2 0's is precisely $\binom{p-q+2}{q}$.

Q3: (34pts)
How many monotone paths are there in a grid from (0,0) to (n, n) which only meet the diagonal at $(0, 0)$ and (n, n)?
How many monotone paths are there in a grid from (0,0) to (n, n) which meet the diagonal exactly once between $(0, 0)$ and (n, n)?