Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics, Fall 1999: Test 1

Name:____________________________________

Please circle your TA's name: Ojas Parekh   Goran Konjevod

Problem	Points	Score
1	32	
2	34	
3	34	
Total	100	

No notes, tables, books, calculators can be used during the test!

Q1: (32pts)

(a) What is the coefficient of $x_1^4 x_2^6 x_3^5$ in $(x_1 + x_2 + x_3)^{15}$?

(b) What is the coefficient of $x_1^4 x_2^6 x_3^5$ in $(x_1 + 2x_2 + 3x_3)^{15}$?

(c) How many ways are there of colouring 10 identical balls with 3 colours?

(d) How many ways are there of colouring 10 distinguishable balls with 3 colours?

Q2: (34pts) Show that the number of k-subsets of $\{1, 2, \ldots, n\}$ that do not contain a pair of integers i, j with $|i - j| \leq 2$ is

$$\binom{n - 2k + 2}{k}.$$

Q3: (34pts)

(a) How many sequences are there of length n made up from symbols $\{a, b, c, d\}$?

(b) By considering the sequences that have k a's for $k = 0, 1, 2, \ldots, n$ show that

$$\sum_{k=0}^{n} \binom{n}{k} 3^{n-k} = 4^n. \tag{1}$$

(c) Deduce (1) directly from the binomial theorem.