Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics: Section 1
Test 1

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts) A word over the alphabet $\{a, b, c, \ldots, z\}$ is called *increasing* if its letters, apart from repetitions, appear in alphabetical order. For example $aabcc$ is increasing, but $abzc$ is not. How many increasing words are there of length n?

Q2: (33pts)

(a) How many sequences are there of length n made up from symbols $\{a, b, c\}$?

(b) By considering the sequences that have k a's for $k = 0, 1, 2, \ldots, n$ show that

$$\sum_{k=0}^{n} \binom{n}{k} 2^{n-k} = 3^n. \tag{1}$$

(c) Deduce (1) directly from the binomial theorem.

Q3: (34pts) In this question n DISTINGUISHABLE 6-sided dice are rolled sequentially.

(a) What is the probability that none of them come up 4,5 or 6?

(b) If Low stands for 1,2 or 3 and High stands 4,5 or 6, what is the probability that there are an even number of times when the dice outcome is Low?

(c) What is the probability that there are never two consecutive rolls which are Low?