1. Let the sequence of integers $M_1, M_2, M_3, \ldots$, be defined by the recurrence relation

\[ M_1 = 3 \text{ and } M_k = kM_{k-1} - k + 2 \quad (k > 1). \]

Show by induction on $k$ that if the edges of $K_{M_k}$ are coloured in $k$ colours then there is a monochromatic triangle.
(Hint: let $V_i$ denote the set of vertices $v$ for which edge $\{1, v\}$ has color $i$.)

2. Let $m = s(p - 1) + 1$ and $n = s^m(q - 1) + 1$. Show that every $s$-coloring of the edges of $K_{m,n}$ using $s$ colors contains a monochromatic copy of $K_{p,q}$.
(Hint: let $X, Y$ be the two parts of the bipartition in $K_{m,n}$. Begin by showing that there must be $q$ vertices $Q \subseteq Y$ such that $c(x, y) = c(x, y')$ for all $x \in X$ if $y, y' \in Q$.)

3. Prove that for $n = 2m$ sufficiently large, every 2-coloring of the edges of $K_{n,n}$ contains a monochromatic copy of $K_{p,p}$.
(Hint: suppose that $X = \{x_1, x_2, \ldots, x_n\}, Y = \{y_1, y_2, \ldots, y_n\}$ are the two parts of the bipartition in $K_{n,n}$. Consider the 2-coloring of $K_m$ induced by the colors of the edges $(x_i, y_{j+m})$ for $i, j \leq m.$)