1. Find the set of $P$-positions for the take-away games with subtraction sets

   (a) $S = \{1, 3, 7\}$.
   (b) $S = \{1, 4, 6\}$.

   Suppose now that there are two piles and the rules for each pile are as above. Now find the $P$ positions for the two pile game where in one pile $S$ is as in (a) and the other pile is as in (b).

2. Consider the following game: There is a pile of $n$ chips. A move consists of removing any proper factor of $n$ chips from the pile. (For the purposes of this question, a proper factor of $n$, is any factor, including 1, that is strictly less than $n$.) The player to leave a pile with one chip wins. Determine the $N$ and $P$ positions and a winning strategy from an $N$ position.

3. In a take-away game, the set $S$ of the possible numbers of chips to remove is finite. Show that the Sprague-Grundy numbers satisfy $g(n) \leq |S|$ where $n$ is the number of chips remaining.