1. Let $\mathcal{M}$ be a matroid (without loops) on $E$ and suppose that $r(E) = \ell$. Suppose that $\ell < k \leq |E|$ and $\mathcal{B}_k$ is the set of $k$-subsets of $E$ that contain a base of $\mathcal{M}$. Show that $\mathcal{B}_k$ forms the set of bases of another matroid.

2. Let $\mathcal{H} = \{H_1, H_2, \ldots, H_m\} \subseteq \binom{E}{k}$. Suppose also that $|H_i \cap H_j| \leq k - 2$ for $i \neq j$. Show that $\mathcal{B}_\mathcal{H} = \binom{E}{k} \setminus \mathcal{H}$ forms the set of bases of a matroid.

3. Let $\mathcal{M}$ be a hereditary system and suppose the the rank function $r$ is submodular. Prove that $\mathcal{M}$ satisfies the Weak Elimination Property for circuits.

(WEP: if $C_1, C_2$ are circuits and $e \in C_1 \cap C_2$ then there exists a circuit $C \subseteq (C_1 \cup C_2) \setminus \{e\}$.)