21-301 Combinatorics
Homework 3
Due: Wednesday, September 30

1. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of **variables**. A **literal** is a variable x_i or its **negation** $\bar{x}_i$. A **clause** C is a set of literals. Let $\mathcal{C} = \{C_1, C_2, \ldots, C_m\}$ be a set of clauses, all of size k. A variable x_i appears in a clause C_j if $\{x_i, \bar{x}_i\} \cap C_j \neq \emptyset$. An **assignment** of truth values is a map $\sigma : X \to \{0, 1\}$. We extend σ to the set of literals L by putting $\sigma(\bar{x}_i) = 1 - \sigma(x_i)$ for $i = 1, 2, \ldots, n$. We say that σ **satisfies** $\mathcal{C}$ if every clause cojntains at least one literal y such that $\sigma(y) = 1$. Show that if no variable appears in more than $2^{k-2}/k$ clauses, then there is a least one satisfying assignment.

2. Let $G = (V, E)$ be a graph and suppose each $v \in V$ is associated with a set $S(v)$ of colors of size at least $10d$, where $d \geq 1$. Suppose that for every v and $c \in S(v)$ there are at most d neighbors u of v such that c lies in $S(u)$. Prove that there is a proper coloring of G assigning to each vertex v a color from its class $S(v)$. (By proper we mean that adjacent vertices get distinct colors.)

3. Suppose that $n \leq 2^{k-3}/k$. Show that $[n]$ can be partitioned into two sets B, W such that neither B nor W contains a k-term arithmetic progression i.e. a set $\{a + ib : i = 0, 1, \ldots, k-1\}$.