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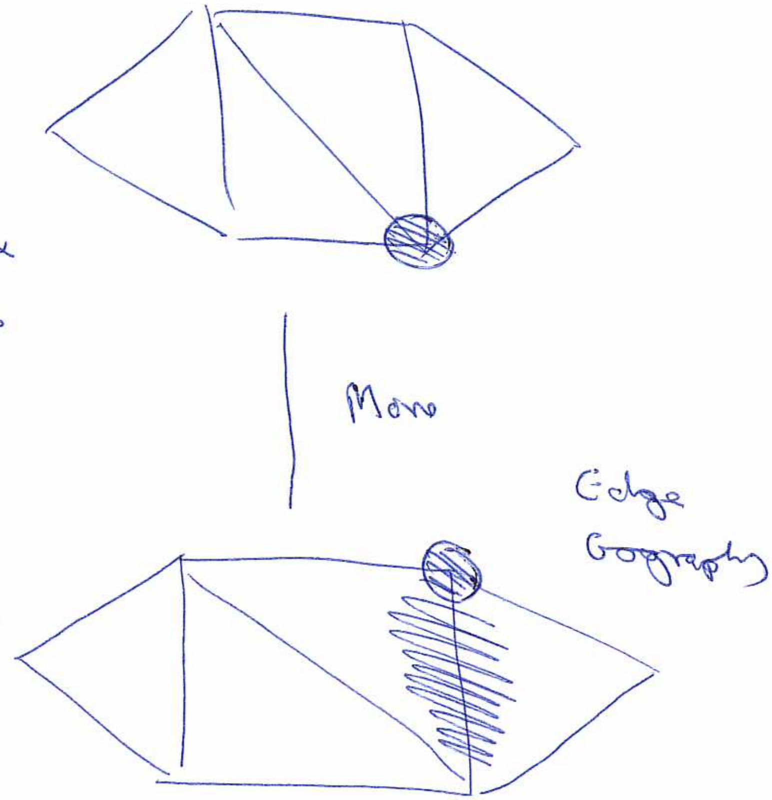
Geography



Start with a chip on a graph vertex x and move the chip to a neighbor y.

Vertex Geography: delete x

Edge Geography: delete y

Graph can be directed or undirected.

Winner = person who make the last move.

Complexity: (Can you tell "quickly" N from P)

Digraph: Edge & vertex geography are PSPACE-hard.

Graphs: Edge geograph is PSPACE-hard (

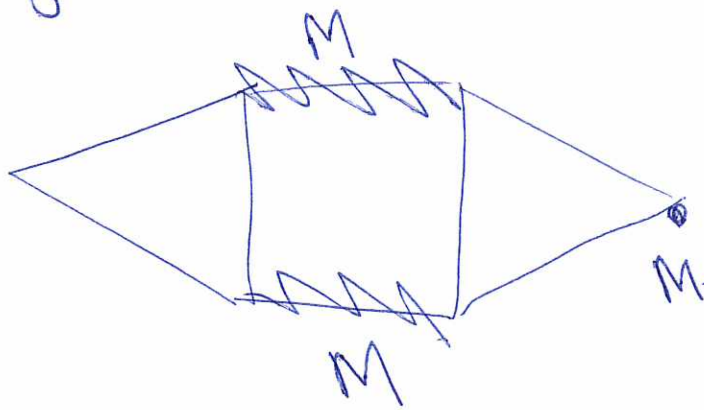
Vertex geography is in P

Edge geography on a bipartite graph is in P]

Vertex Geograph in (undirected) graphs

Need some simple observations about matchings.

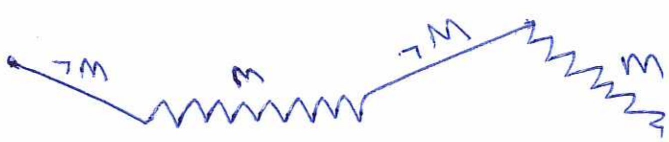
Matching M = {disjoint edges}

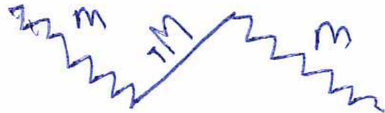


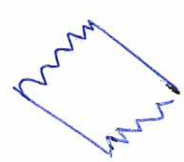
M is perfect if no vertex is M-unsaturated.

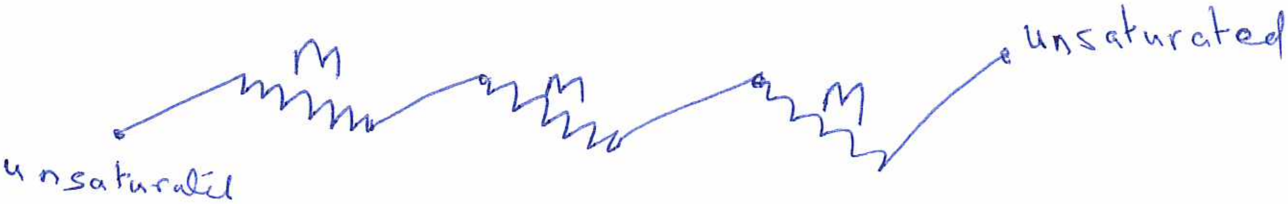
Alternating Paths

A path is M-alternating if the edges go $M, \neg M, M, \neg M$, etc









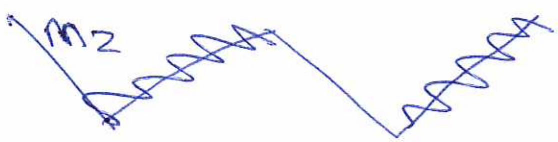
Lemma

M is a maximum size matching iff there are no M-augmenting path.

Proof

If there is an augmenting path we remove its M-edge from M and add the $\neg M$ edge to get a matching one size bigger.

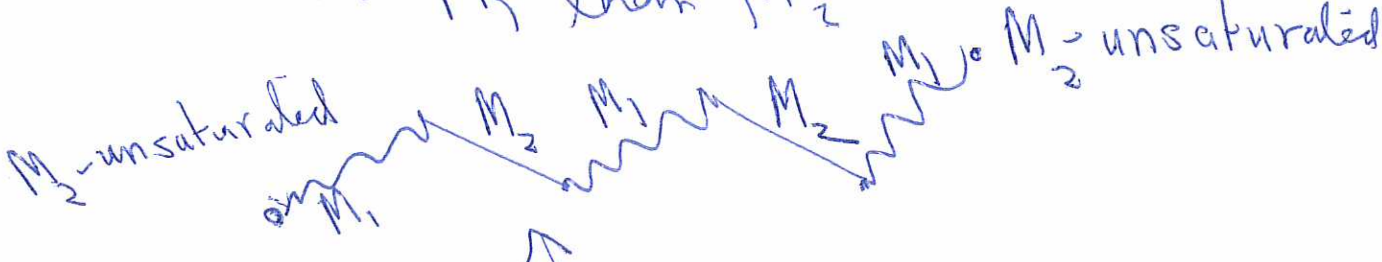
Suppose M_1 & M_2 are matchings & $|M_1| > |M_2|$

$M_1 \oplus M_2$: 

= alternating paths & cycle.

$M_1 \setminus M_2 \cup M_2 \setminus M_1$

If $|M_1| > |M_2|$ then at least one path has more M_1 than M_2



↑ M_2 - augmenting

Geography

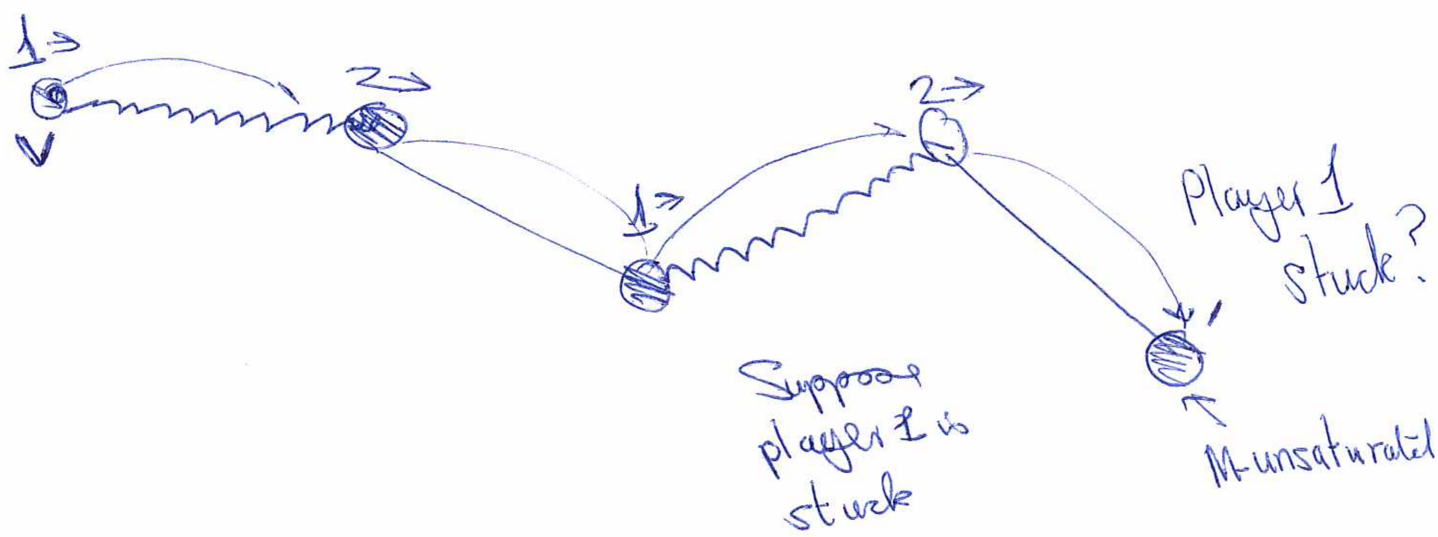
(G, v) is an N-position iff every maximum matching in G, covers v.

Proof

Suppose v is in every maximum matching.

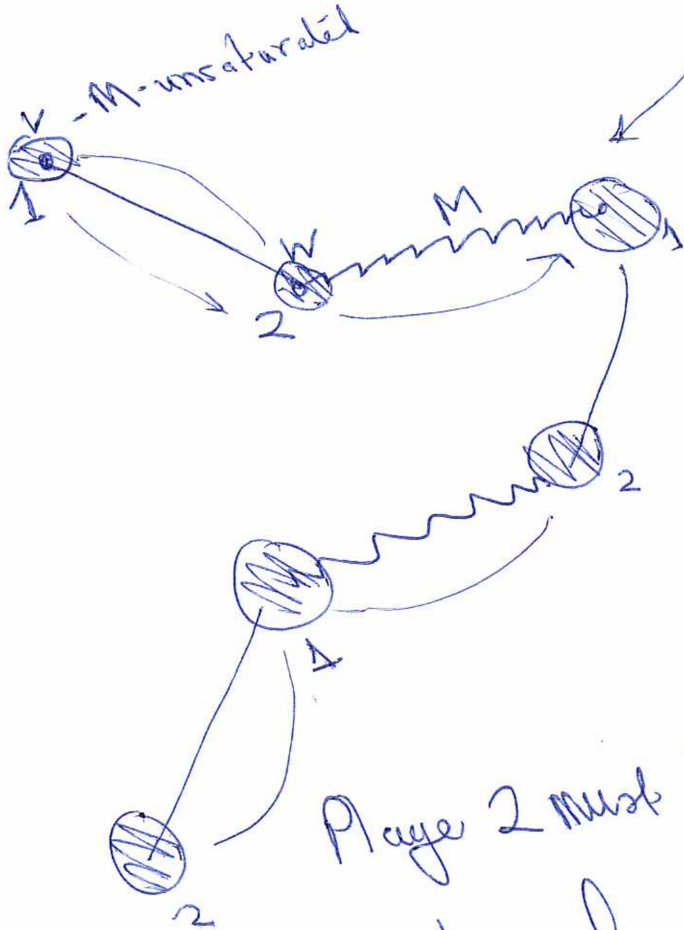
Player 1 chooses some maximum matching M.

Moves along M-edge



Alternating path give a matching M′ s.t.:
|M′| = |M| & M′ doesn't cover v. - contradiction

Suppose next that $\exists$ maximum matching M which does not cover v.



exists else w is M unsaturated and $M + \{v,w\}$ is a bigger matching than M.

Player 2 must have a move else we have found an augmenting path i.e. M is not maximum.

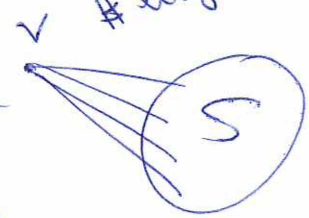
Undirected Edge Geography – Bipartite Graph.

~~Defn~~ G is a graph, S is an <u>even</u> <u>kernel</u> if

(i) S is an independent set i.e. no edge is contained in S

(ii) $v \notin S$ then $\deg_S(v)$ is even

\# edge to S



Lemma

If S is an even kernel and $v \in S$ then (G, v) is a P-position.

Proof