10/9/15

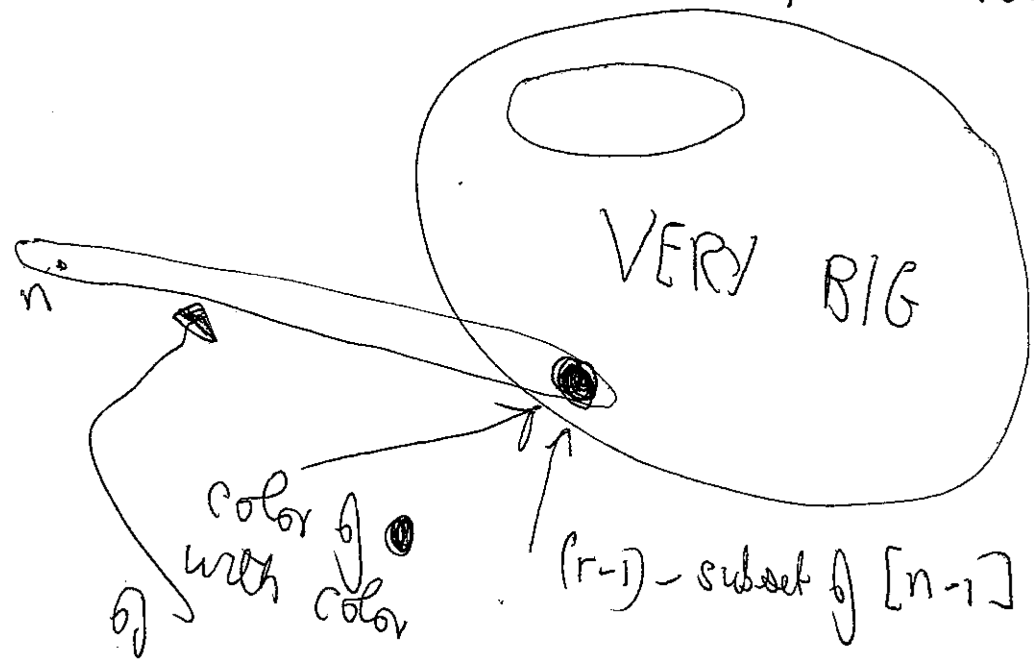
$\exists N(p,q;r)$ such that if $n \geq N(p,q;r)$ and we color $\binom{[n]}{r}$ Red/Blue then there exists $S \subseteq [n]$, $|S| = p$ and $\binom{S}{r}$ is Red OR exists $T \subseteq [n]$, $|T| = q$ and $\binom{T}{r}$ is Blue.

We proved this for $r=2$.

Assume inductively that claim is true for $r-1$ and r with $p'+q' < p+q$.

Assume $r \geq 3$

$n = M+1$, M very big



M is big enough so that either there is a set S of size $p_1 = N(p-1, q; r)$ such that $\binom{S}{r-1}$ is Red or there is T of size $q_1 = N(p, q-1; r)$ such that $\binom{T}{r-1}$ is Blue.

Assume S exists.

Either there is a Blue S' of size q ✓

or there is a Red S'' of size $p-1$ i.e. $\binom{S''}{r}$ is Red

~~Th~~ Then $S'' \cup \{n\}$ is a Red set of size p

r-subset of $S'' \cup \{n\}$

is either $\subseteq S''$ ✓ since $\binom{S''}{r}$ is Red

r-subset $= A \cup \{n\}$, $|A| = r-1$, $A \in \binom{S}{r-1}$ is Red.

Applications

Schur's Theorem

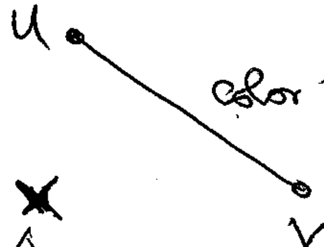
$r_k = N(\underbrace{3,3,\cdots 3}_{k}; 2)$ = smallest n such that if we k-color the edges of K_n then there exists a mono-chromatic $\triangle$.

Theorem

For all partitions of $[r_k]$ there exist i and $x, y, z \in S_i$ such that $x + y = z$.

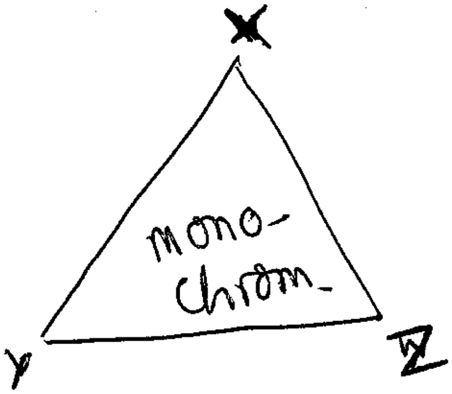
Proof: Need to color edges of K_n

$n \geq r_k$.



color i where $|u - v| \in S_i$

$x < y < z$

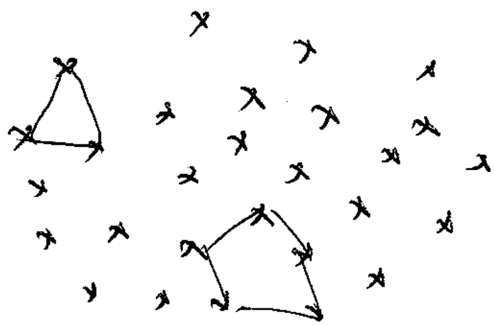


$u = y - x \in S_i$

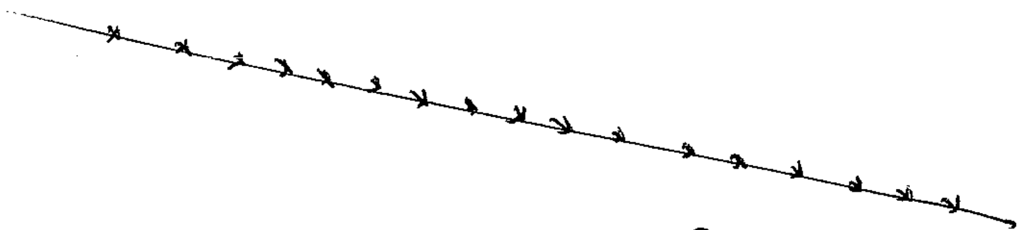
$v = z - y \in S_i$

$w = z - x \in S_i$

$u + v = w$



Can you find a large polygon which is convex

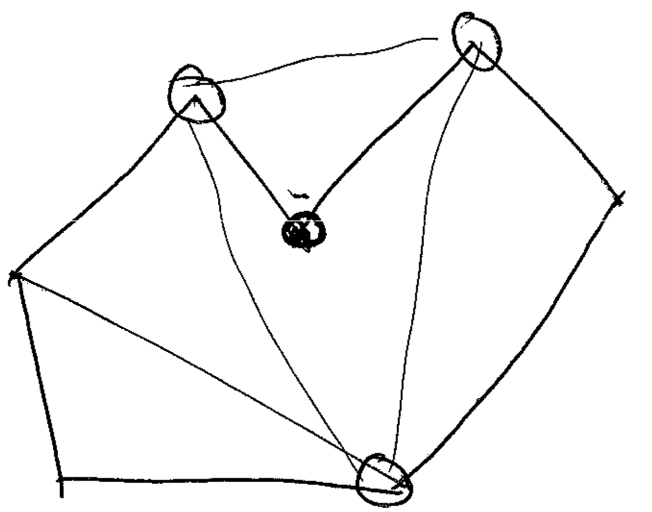


A set of points in $\mathbb{R}^2$ is in general position if no 3 points are collinear.

__Theorem__

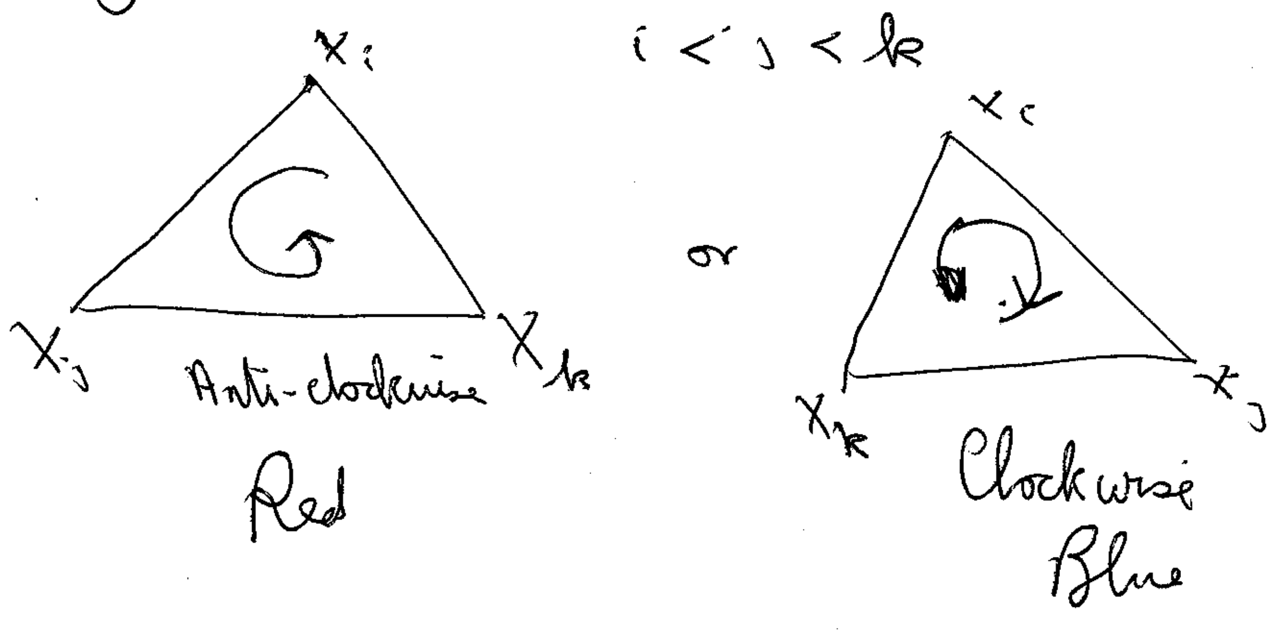
if $n \geq N(k,k;3)$ and X is a set of n points in $\mathbb{R}^2$ in general position then X contains a k-subset Y that forms a convex polygon.

If every 4 points of a set Y are convex then Y is convex.



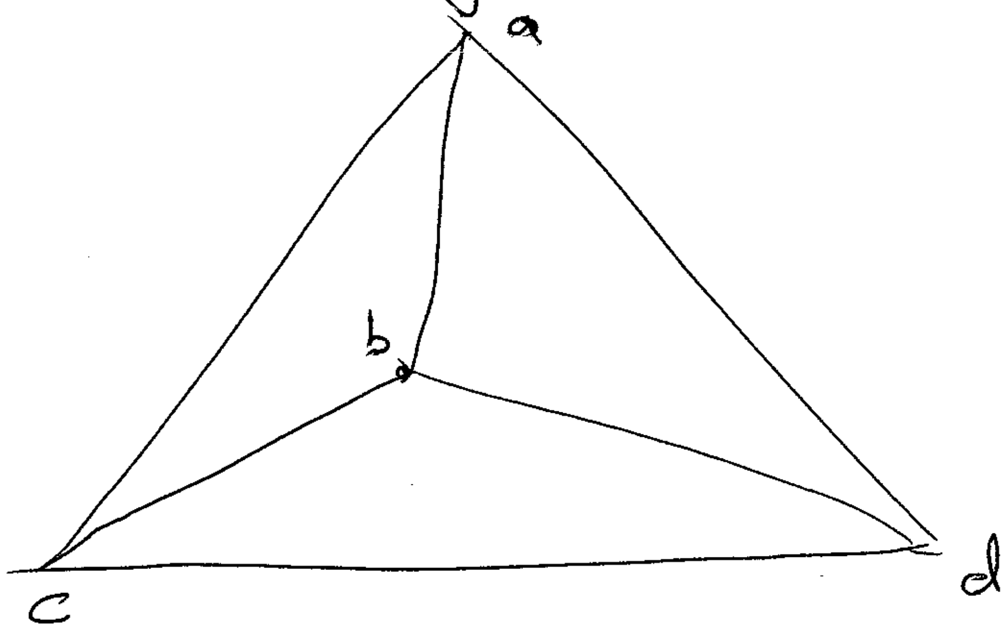
We must show that there exists a k-set Y such that all 4-subsets of Y are convex.

Coloring: label points of X as $X_1, X_2, \ldots, X_n$



So there exist Y such that every $\triangle$ in Y has same color.

Claim: Y defines a convex polygon.



Case 1

$a < b < c < d$

abc is Blue
abd is Red
X

Case 2

$a < b < d < c$

abc is blue
abd is Red
X

Case 3

$a < c < b < d$

acb is Red
abd is Red
acd is Red
bcd is Blue
X

~~We can replace~~

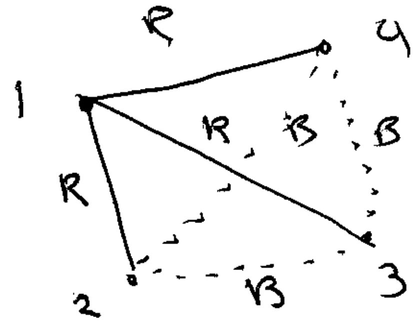
Define $r(H_1, H_2)$ is minimum n such that in any 2-coloring of K_n there is a Red H_1 or a Blue H_2.

We have dealt with case where H_1, H_2 are complete

Claim: $R(P_3, P_3) = 5$ $\quad$ $[R(\Delta, \Delta) = 6$

$R(\wedge, \wedge)$

$R(P_3, P_3) > 4$



$R(P_3, P_3) \leq 5.$