21-301 Combinatorics
Homework 8
Due: Monday, November 19

1. In a take-away game, the set S of the possible numbers of chips to remove is finite. Show that the Grundy numbers g satisfy $g(n) \leq |S|$ where n is the number of chips remaining.

2. Consider the following take-away game: In the first move you are not allowed to take the whole pile. After that, if a player removes x chips, then the next player can remove up to $\lfloor 5x/4 \rfloor$ chips. Determine the P positions.

3. Find the set of P-positions for the take-away games with subtraction sets

   (a) $S = \{1, 3, 7\}$.

   (b) $S = \{1, 4, 6\}$.

   Suppose now that there are two piles and the rules for each pile are as above. Now find the P positions for the two pile game.