

21-301 Combinatorics  
Homework 7  
Due: Monday, November 12

1. Let  $r_n = r(3, 3, \dots, 3)$  be the minimum integer such that if we  $n$ -color the edges of the complete graph  $K_N$  there is a monochromatic triangle.
  - (a) Show that  $r_n \leq n(r_{n-1} - 1) + 2$ .
  - (b) Using  $r_2 = 6$ , show that  $r_n \leq \lfloor n!e \rfloor + 1$ .
2. Show that if the edges of  $K_{m+n}$  are colored red and blue then either (i) there is a red path with  $m$  edges or (ii) a vertex of blue degree at least  $n$ .
3. Given a set  $I$  of  $n$  intervals in  $R$ , assume that there is no nested set of intervals with size  $k$  (a set of intervals are nested if for every pair, one is completely contained inside the other). Then prove that there exists a subset of size  $\lceil n/k \rceil$  where no pair of intervals are nested.