

21-301 Combinatorics
Homework 6
Due: Monday, November 5

1. Let G be a bipartite graph with bipartition A, B where $|A| = |B| = n$. Let m be the number of edges of G . Show that if $r \geq 2$ and

$$n \binom{m/n}{2} > (r-1) \binom{n}{2}$$

then G contains a copy of $K_{2,r}$. Here $K_{2,r}$ is a bipartite graph with vertex set X, Y where $|X| = 2, |Y| = r$ and edge set $X \times Y$.

2. Use the pigeon-hole principle to show that for every integer $k \geq 1$ and prime $p \neq 2, 5$ there exists a power of p that ends with $000 \cdots 0001$ (k 0's).
3. Suppose we 2-color the edges of K_6 Red and Blue. Show that there are at least two monochromatic triangles.