

Homework 4

Due Wednesday, October 10th,

1. In the kingdom of Far Far Away there are coins of values 1, 2 and 3 dollars. In how many ways can the people of Far Far Away change n dollars?

Hint:

$$\frac{1}{1+x+x^2} = \sum_{n=0}^{\infty} \left(\left(\frac{1}{2} + \frac{\sqrt{3}i}{6} \right) \left(-\frac{1}{2} - \frac{\sqrt{3}i}{2} \right)^n + \left(\frac{1}{2} - \frac{\sqrt{3}i}{6} \right) \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right)^n \right) x^n.$$

2. Assume you are given n *labeled* bills of value 1 dollar, m labeled bills of value 2 dollars and p labeled bills of value 3 dollars. Write the generating function for the number of ways in which you can change N dollars given these labeled bills. You do not have to write the generating function as a power series.
3. Let $B(n)$ be the number of ways in which one can put n labeled balls into unlabeled sacks. For instance balls $\{1, 2, 3\}$ could be split in five ways:

$$\{\{1\}, \{2\}, \{3\}\}, \{\{1, 2\}, \{3\}\}, \{\{1, 3\}, \{2\}\}, \{\{1\}, \{2, 3\}\}, \{\{1, 2, 3\}\}.$$

Find a recursion for $B(n)$. You do not need to solve the recursion.