21-301 Combinatorics
Homework 2
Due: Friday, September 14

1. Show that for any $n \geq 0$
$$\sum_{0 \leq i \leq k \leq n} \binom{n}{k}\binom{k}{i} = 3^n,$$
where the sum goes over all integer pairs i, k such that $0 \leq i \leq k \leq n$.

2. In class we have defined the *Catalan number* C_n to be $|\mathrm{PATHS}_{\geq}(\mathrm{n}, \mathrm{n})|$ and showed that $C_n = \frac{1}{n+1}\binom{2n}{n}$. Prove that
$$C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}.$$

3. In how many ways can $2n$ people sitting in a cycle shake hands without crossing arms?

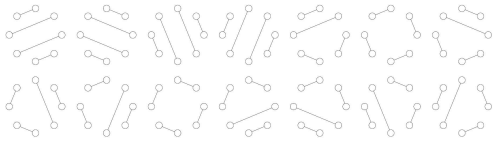

Figure 1: All handshake configurations for 8 people