21-301 Combinatorics
Homework 1
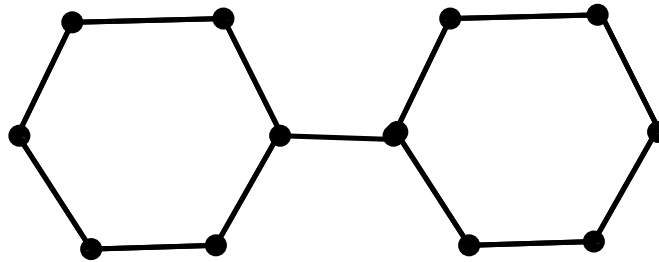## Due: Wednesday, September 5

1. How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 + x_5 = 100$$

   satisfy $x_1 \geq 6$, $x_2 \geq 10$, $x_3 \geq -3$, $x_4 \geq 4$ and $x_5 \geq 4$?

2. Show that

$$\sum_{k=0}^{n} \binom{n}{k}\binom{k}{\ell} = \binom{n}{\ell} 2^{n-\ell}.$$

3. How many ways are there of placing k 1's and $2n - k$ 0's at the vertices of the cycles in the diagram below so that each 1 is separated by at least one 0?

   Each cycle has n vertices and so there are $2n$ vertices altogether.