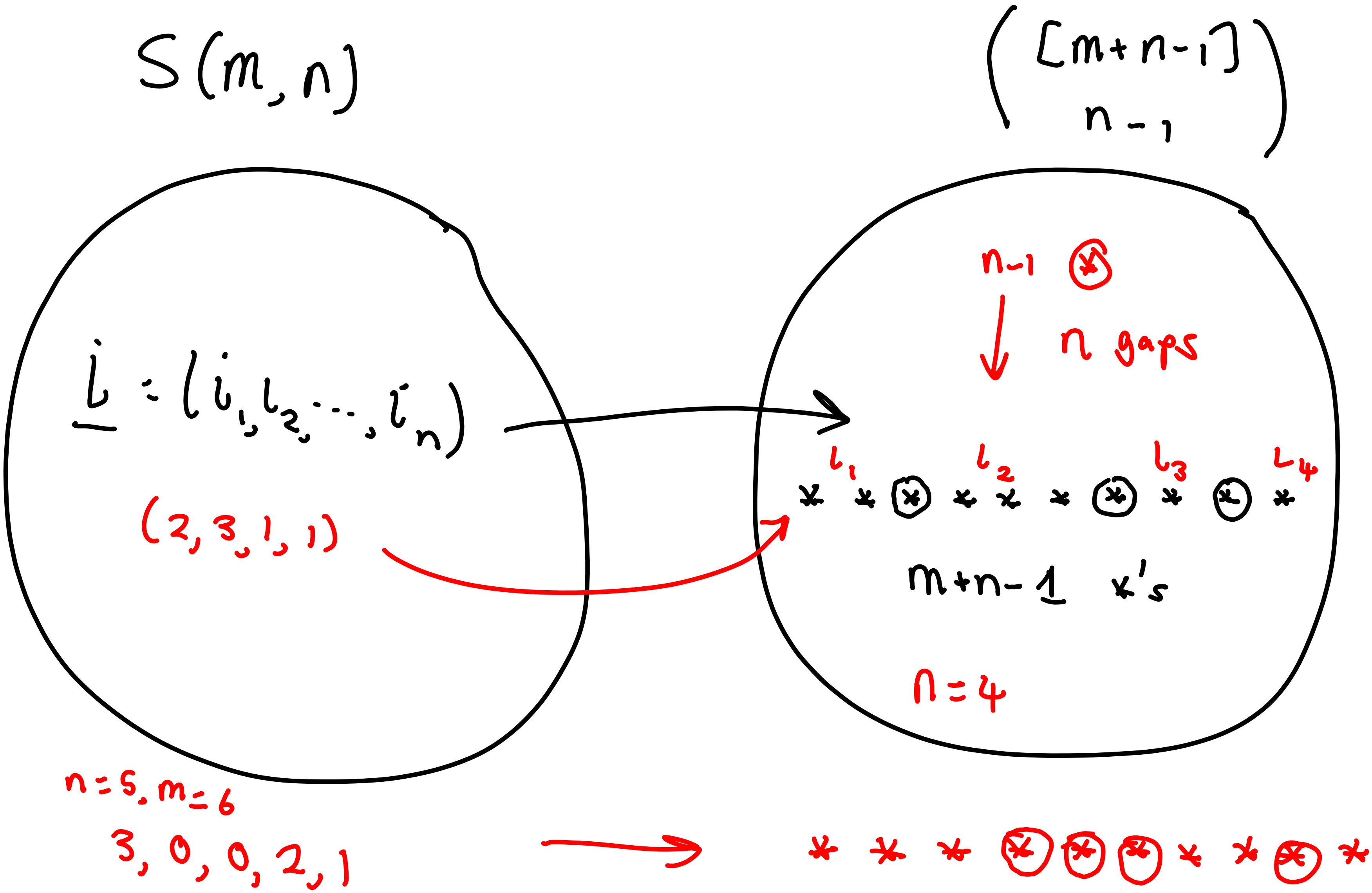
8/29/12

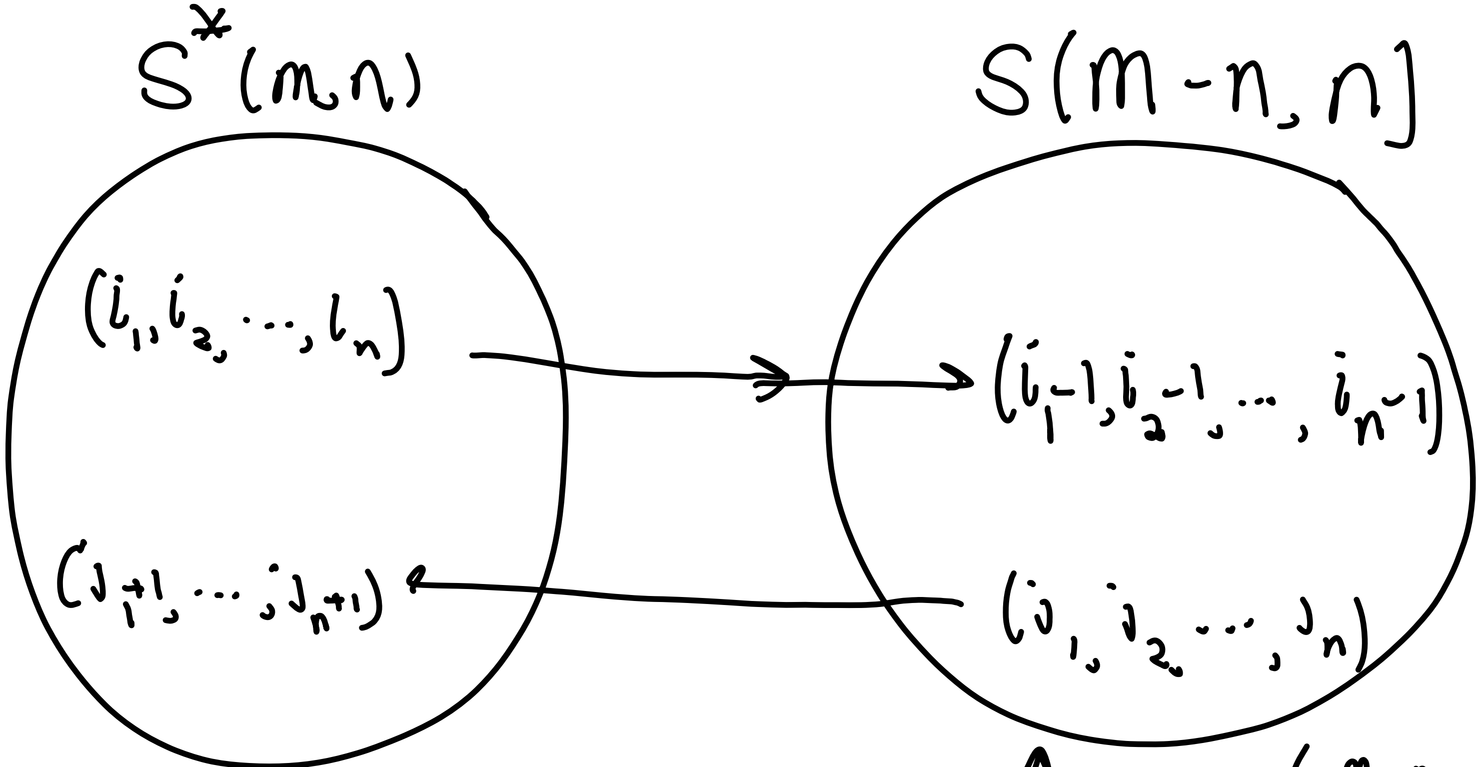
$$S(m,n) = \left\{ (i_1, i_2, \ldots, i_n) : \begin{array}{l} i_1 + i_2 + \cdots + i_n = m \\ i_j \in \{0,1,2,\ldots\} \end{array} \right\}$$

$$|S(m,n)| = \binom{m+n-1}{n-1}$$

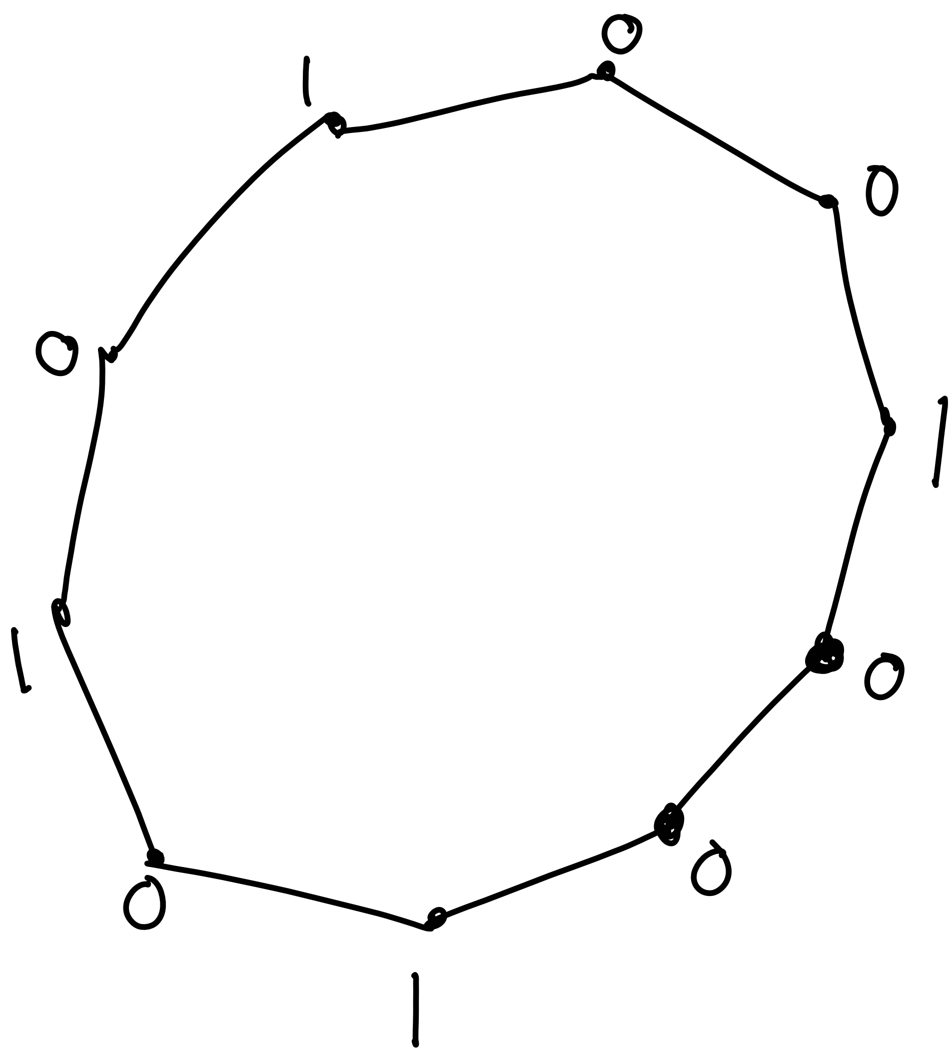


S(m, n)
$\binom{m+n-1}{n-1}$
$\underline{i} = (i_1, i_2, \ldots, i_n)$
(2, 3, 1, 1)
n-1 ⊛
n gaps
i_1 i_2 i_3 i_4
* * ⊛ * * * ⊛ * ⊛ *
m+n-1 *'s
n = 4
n = 5, m = 6
3, 0, 0, 2, 1
* * * ⊛ ⊛ ⊛ * * ⊛ *

$$S^*(m,n) = \{ (i_1, i_2, \ldots, i_n) \in S(m,n) : i_k \geq 1, \ \forall k \}$$



size $= \binom{m-n+n-1}{n-1} = \binom{m-1}{n-1}$



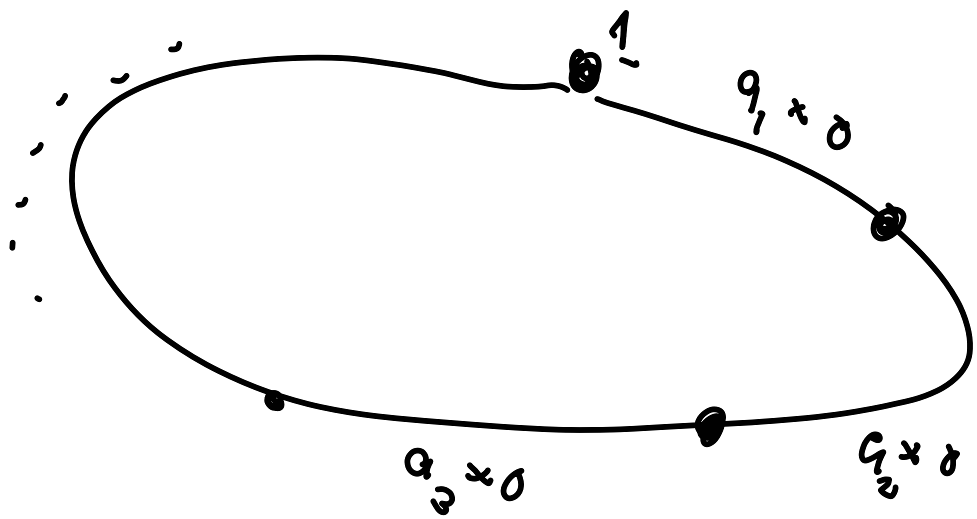
n vertices

How many ways are there of placing 0's & 1's so that no two 1's are adjacent?

Strategy:

(I) Put a $\underline{1}$ somewhere: n choices

(II) Place rest of the $\underline{1}$'s:



$a_1, a_2, \dots, a_k \geq \underline{1}$

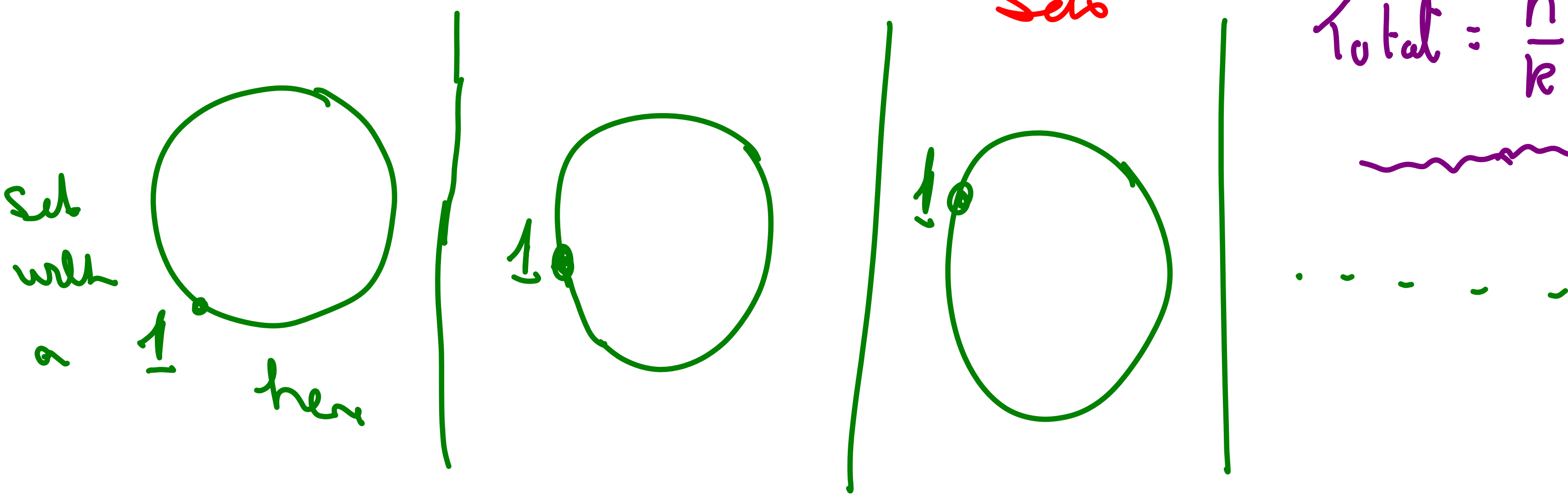
$a_1 + a_2 + \dots + a_k = n-k$

choices for $\underline{a}$: $\binom{n-k-1}{k-1}$

(iii) Every pattern is chosen exactly k times

each pattern occurs in k of these sets

Total $= \dfrac{n}{k}\dbinom{n-k-1}{k-1}$



Identities involving binomial coefficients:

$$\binom{n}{r} = \binom{n}{n-r} \qquad \frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!\,(n-(n-r))!}$$

choose r $\equiv$ choose $n-r$
not to choose

Pascal's Triangle

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

all $(k+1)$-subsets which contain $n+1$

all $(k+1)$-sets that dont contain $n+1$

all $(k+1)$-subsets of $[n+1]$