8/27

Mappings

$$\#\text{ mappings from } [n] \to [m] = m^n$$

$\{1, 2, 3, \ldots, n\}$

$= \#$ sequence $a_1 a_2 \ldots a_n$ where $a_i \in [m]$

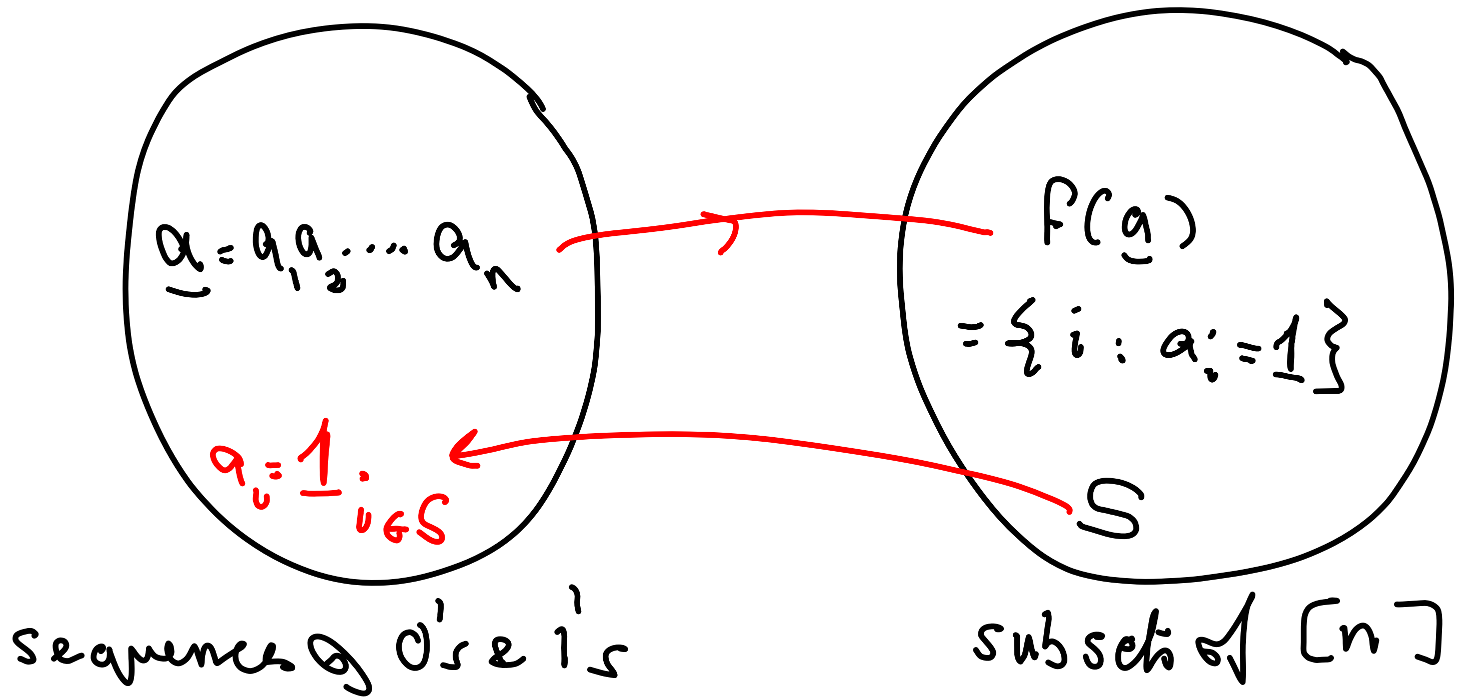
Choose mapping f.

$f(1) \qquad f(2) \qquad \ldots \qquad f(n)$

$$(m \text{ choices}) \times (m \text{ choices}) \times \cdots \times (m \text{ choices}) = m^n$$

$\psi(n) = \#$ subsets of $[n]$

$= 2^n$



# of odd cardinality subsets of $[n]$

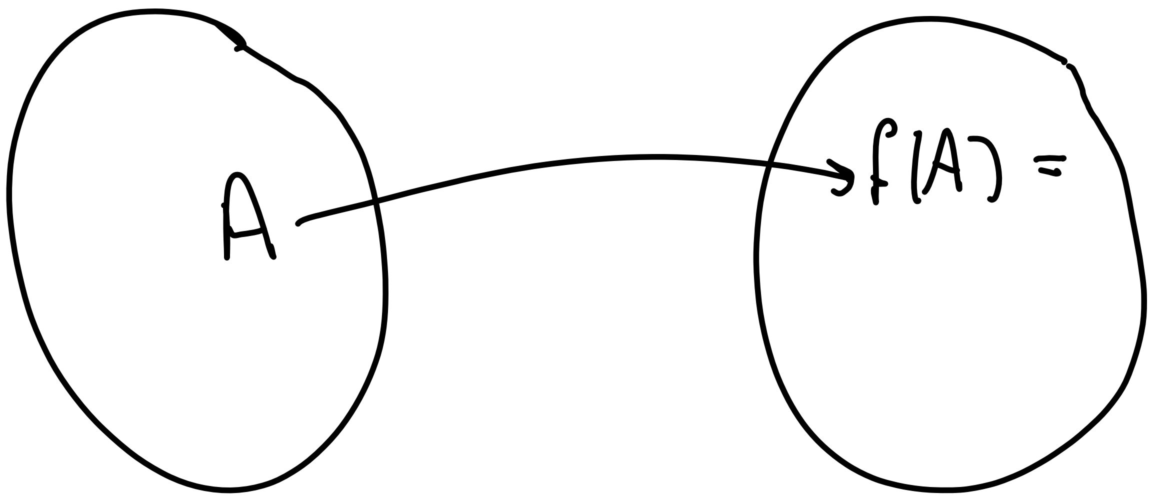
$= \#$ of even cardinality " " "

$= 2^{n-1}$

n is odd: S $\quad$ $\bar{S}$ $\quad$ $\bar{S} = \{x \notin S\}$

odd $\quad$ even

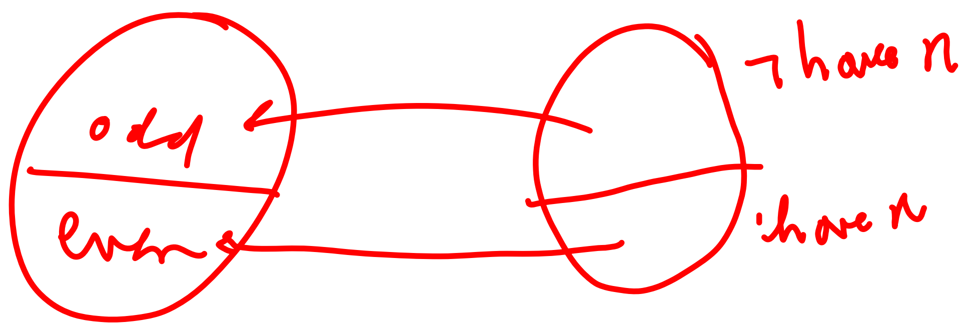
n even?



$$f(A) = \begin{cases} A & \text{if } A \text{ is odd} \\ A + n & \text{if } A \text{ is even} \end{cases}$$

all subsets of $[n-1]$

odd subsets of $[n]$



m^n functions from $[n] \longrightarrow [m]$

$m = n$ permutation of $[n]$

How many are 1-1?

$= m(m-1)\dots(m-n+1)$

$f(1)$ $f(2)$ $f(3)$

$\downarrow$ $\downarrow$ $\downarrow$

(m choices) x (m-1 choices) x (m-2) choices

$=$ # sequence $a_1 a_2 \cdots a_n$

where $a_i \neq a_j$

$i \neq j$

How many are onto?

Inclusion-Exclusion

Binomial Coefficients

X is a finite set.

$$\binom{X}{k} = \{\text{subsets of } X \text{ of size } k\}$$

$$\left|\binom{X}{k}\right| = \binom{|X|}{k} \quad \text{where } \binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

$|\ldots| \equiv \text{size}$

$$\left|\binom{X}{k}\right| k! = |X|(|X|-1)(|X|-2)\cdots(|X|-k+1)$$

choose a set of size k

order it

= choose a sequence of length k, with distinct items.