10\31\12

General Ramsey: $r=3$, $s=2$

Here we could color
$\{1,2,3\}$ with Red
$\{1,2,4\}$ with Blue

Color the triples of $[n]$ with two colors.

Claim: $\forall p, q$, $\exists\, N(p,q;3)$
such that if $n \geq N(p,q;3)$
and we 2-color $[n]^{(3)} = \{ S \subseteq [n] : |S| = 3 \}$
then either $\exists$ a set $X \subseteq [n]$, $|X| = p$, $X^{(3)}$ is Red
or $\exists$ a set $Y \subseteq [n]$, $|Y| = q$, $Y^{(3)}$ is Blue

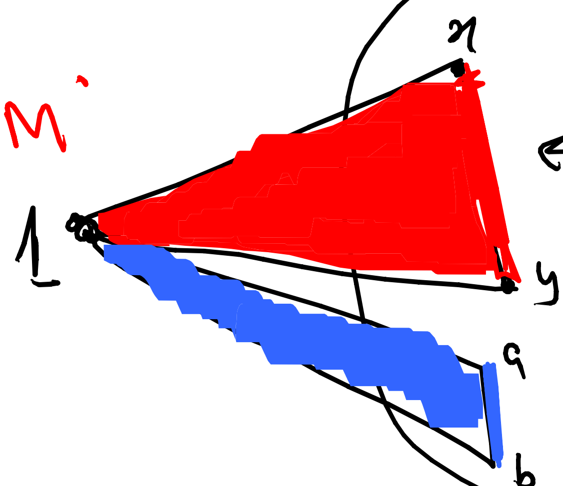
Claim

Inductively exist

$$N(p,q;3) \le 1 + N\Big(\underbrace{N(p-1,q;3)}_{p_1},\ \underbrace{N(p,q-1;3)}_{q_1},\ 2\Big)$$

$$M = R(p_1, q_1)$$

A 2-coloring of $N^{(3)}$ induces a 2-coloring of K_M.



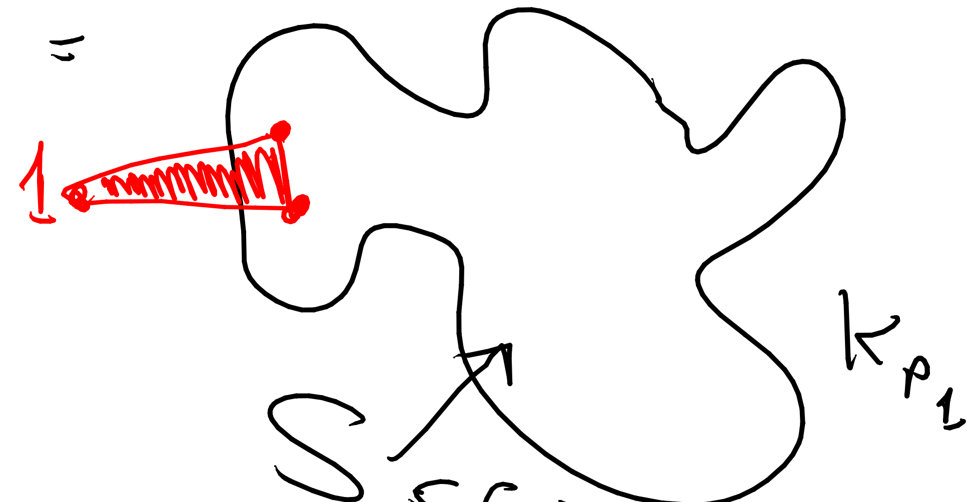
If $1xy$ is Red then we color xy Red

If $1ab$ is Blue then we color ab Blue

$M = R(p_1, q_1) \Rightarrow \exists$ either (i) Red K_{p_1}

or (ii) Blue K_{q_1}

Assume $\exists$ a Red K_{p_1} =



$S \subseteq [n],\ |S| = p_1$

$p_1 = N(p-1, q; 3)$

Now we look at colors of triples of S.

Either (a) blue q-set X

or (b) red $(p-1)$-set Y ← All triples of $Y + \{1\}$ are Red

Schur's Theorem

Let $r_k = N(\underbrace{3,3,3,\ldots,3}_{k};2)$

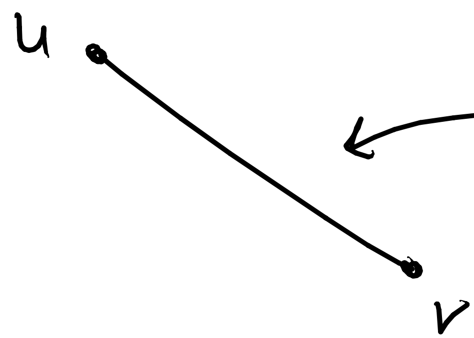
= smallest n such if we color edges of K_n with k colors then $\exists$ a mono-colored triangle.

Theorem

For all partitions $S_1, S_2, \ldots, S_k$ of $[r_k]$ there exists i and $a,b,c \in S_i$ s.t. $a+b=c$

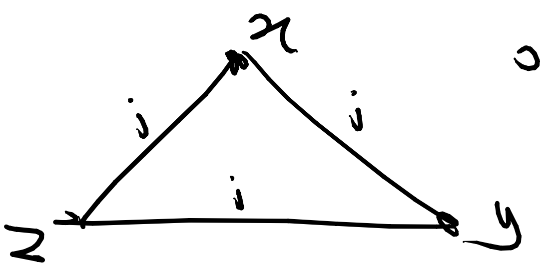
Given $n \geq r_k$ and a partition $S_1, S_2, \dots, S_k$ of $[n]$

Define a coloring of K_n



color i when $|u - v| \in S_i$

$\exists$ a mono-colored triangle

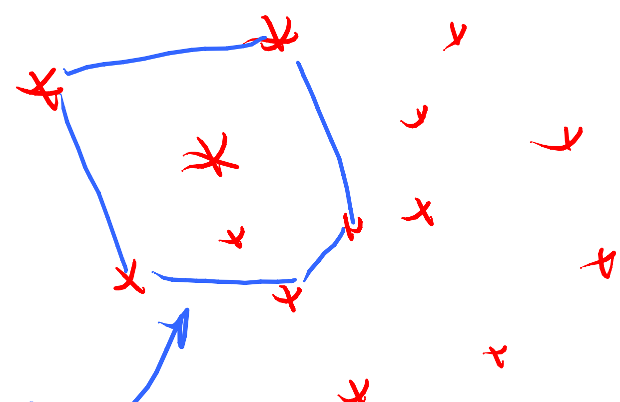


$x < y < z$

$$\Rightarrow \quad \begin{aligned} a &= y - x \in S_i \\ b &= z - y \in S_i \\ \hline c &= z - x \in S_i \end{aligned}$$

Thm

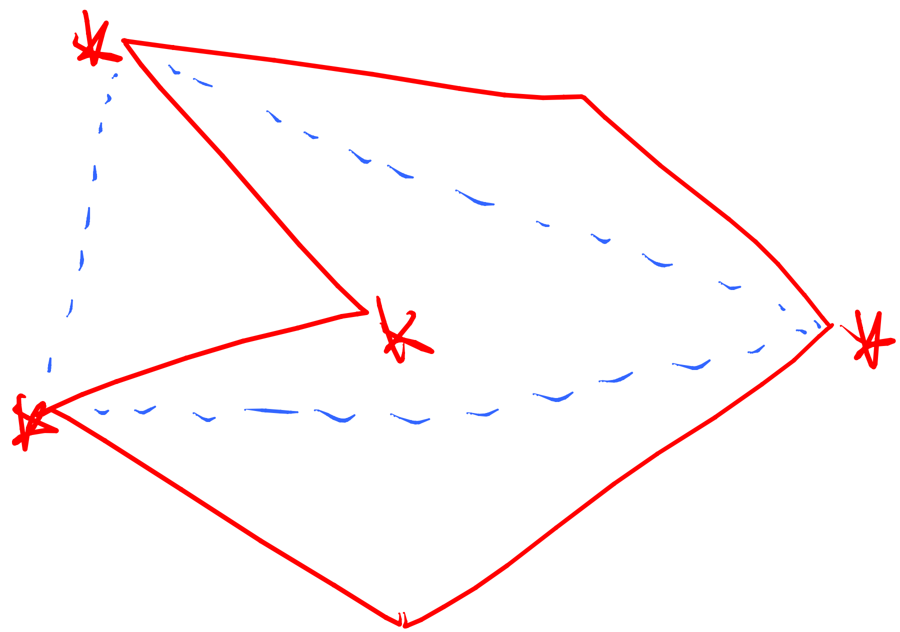
If $n \geq N(k,k;3)$ and X is a set of points in the plane with no 3 in a line then X contains a k-subset Y that are the vertices of a convex polygon.



5 points form a convex polygon

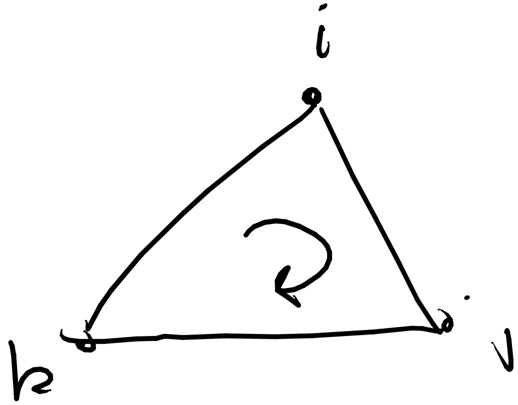
Observation

If every 4-subset $Y \subseteq X$ forms a convex quadrilateral then X forms a convex polygon.



We label the points in X as $X_1, X_2, \ldots, X_n$ and color triples as follows:

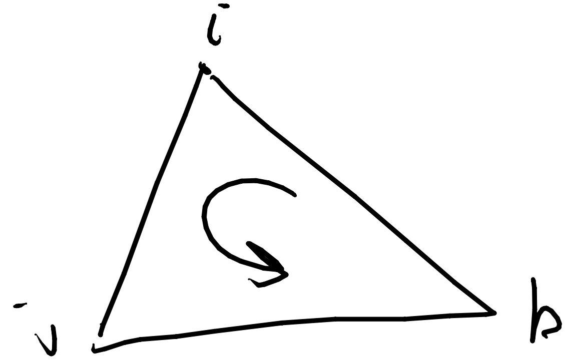
We want to color $X_i X_j X_k$ where $i < j < k$



$i \to j \to k$

clockwise

Triple color is Red



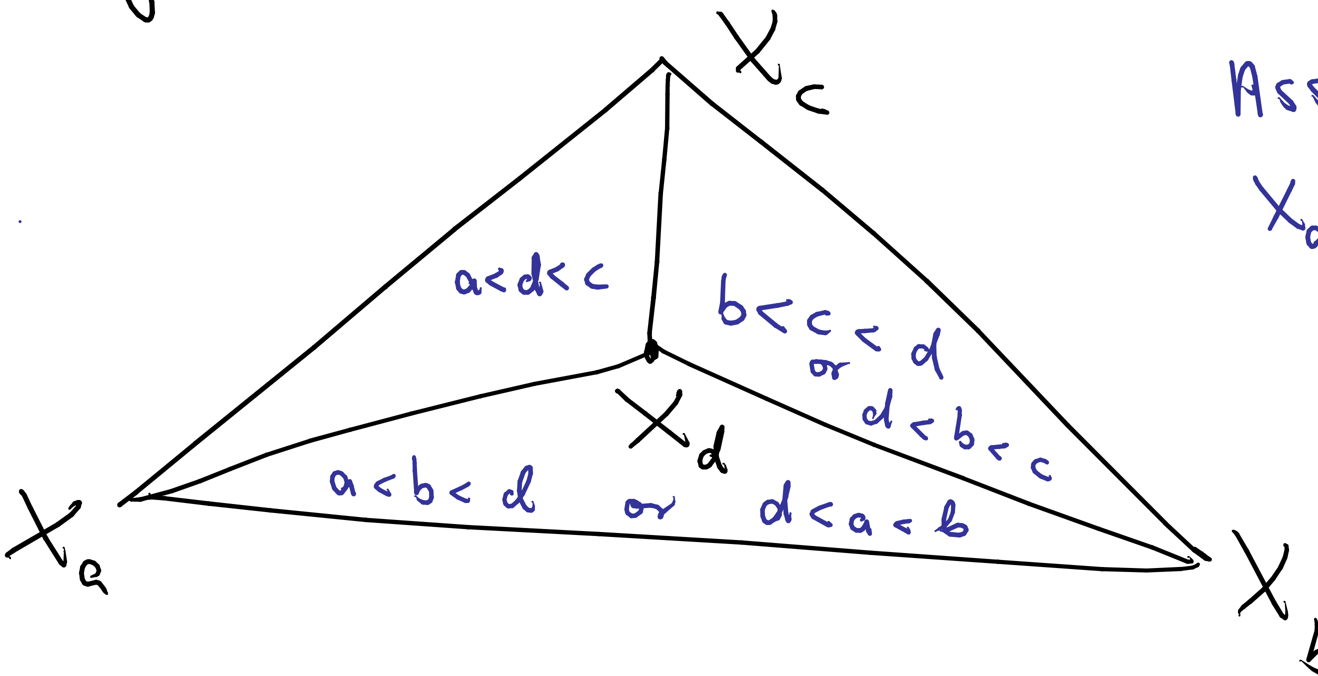
$i \to j \to k$

counter clockwise

Triple Color is Blue

There is a k-set Y in which all triples have same color.

We argue that it does not contain



Assume $a<b<c$

$X_aX_bX_c$ is blue

$\Rightarrow$ Y must form a convex polygon