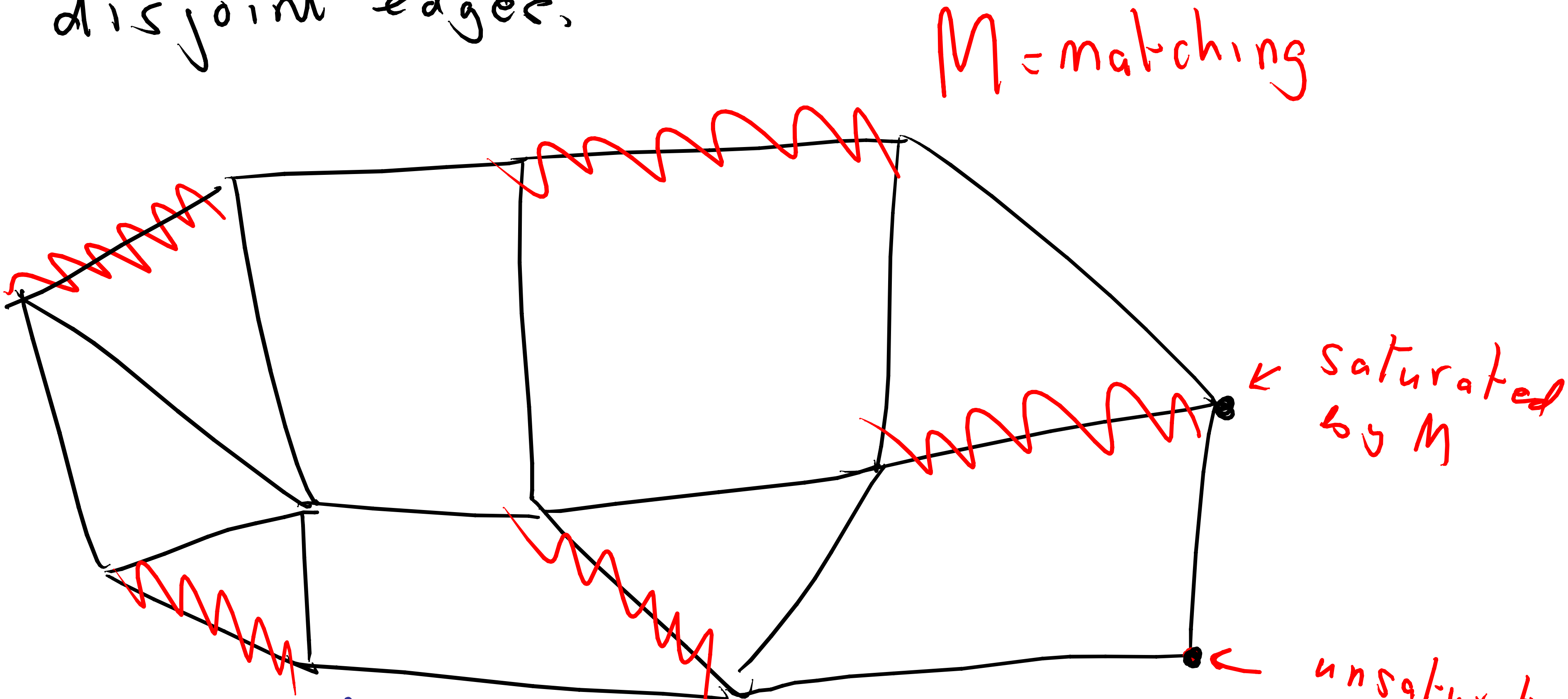
10/22/12

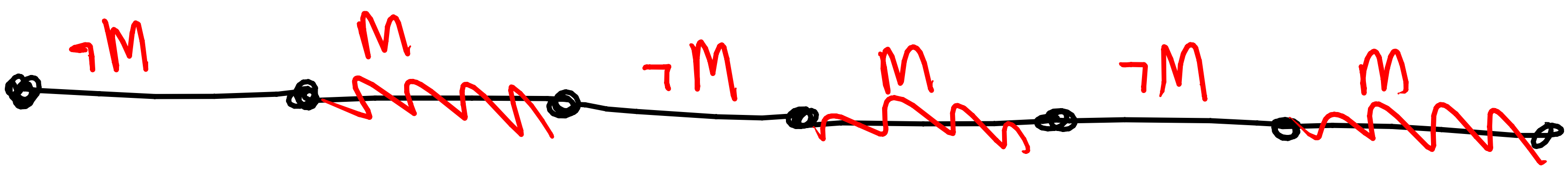
A matching in a graph G is a set of disjoint edges.



M is perfect if every vertex is saturated.

M-alternating path

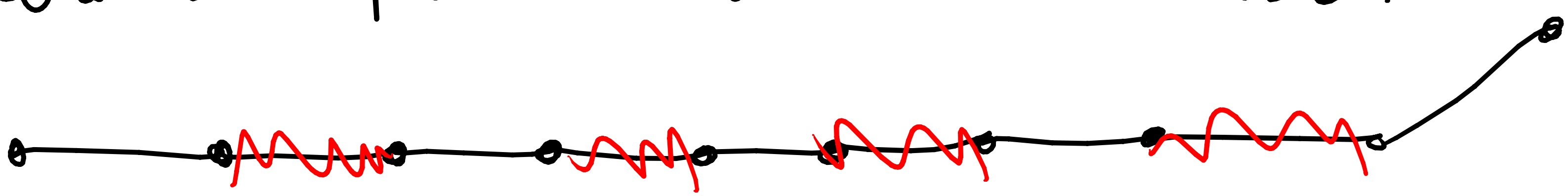
M is a matching.



Edges alternate between M & $\neg M$.

An alternating path is <u>augmenting</u>

if the endpoints are unsaturated.

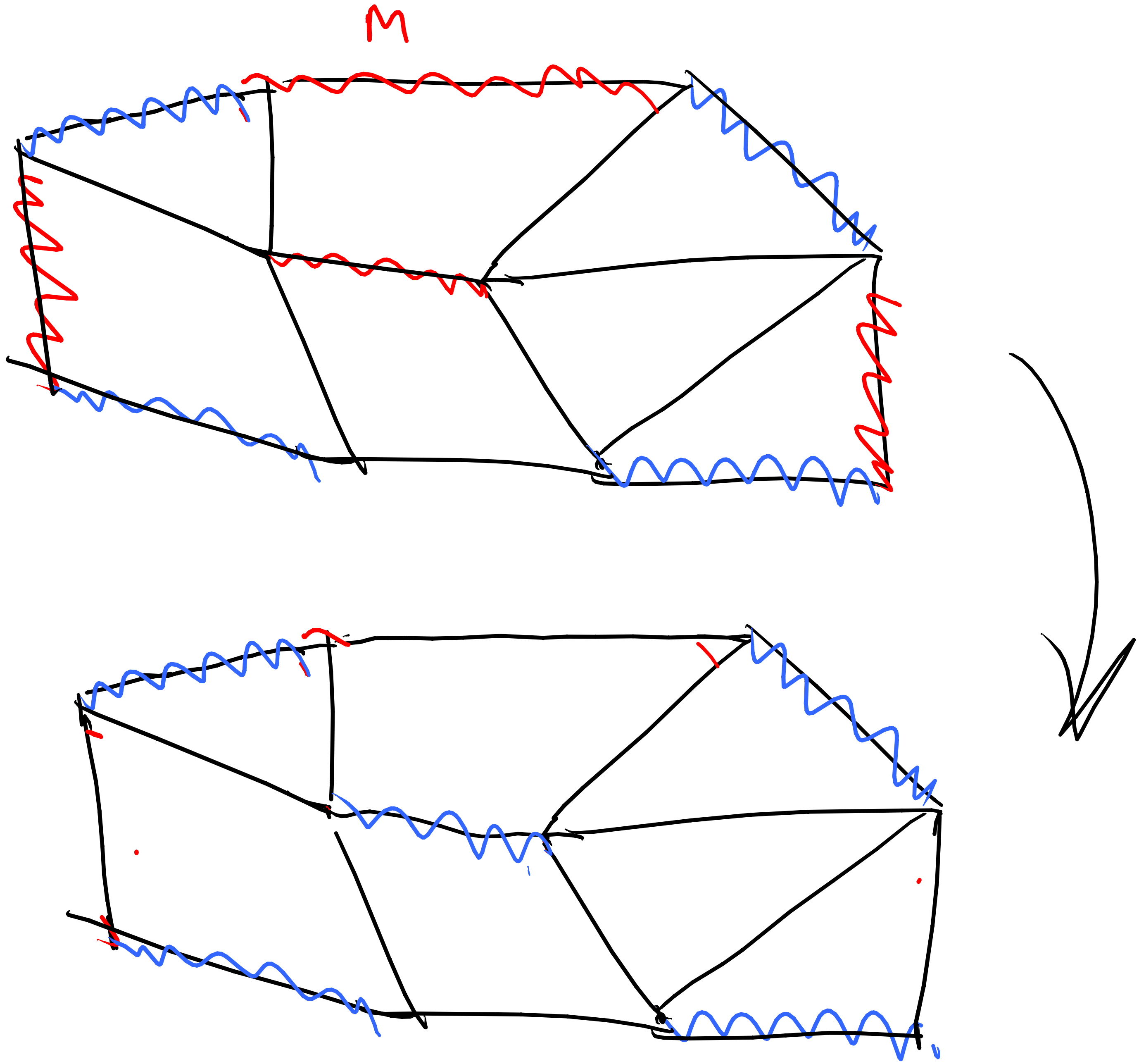


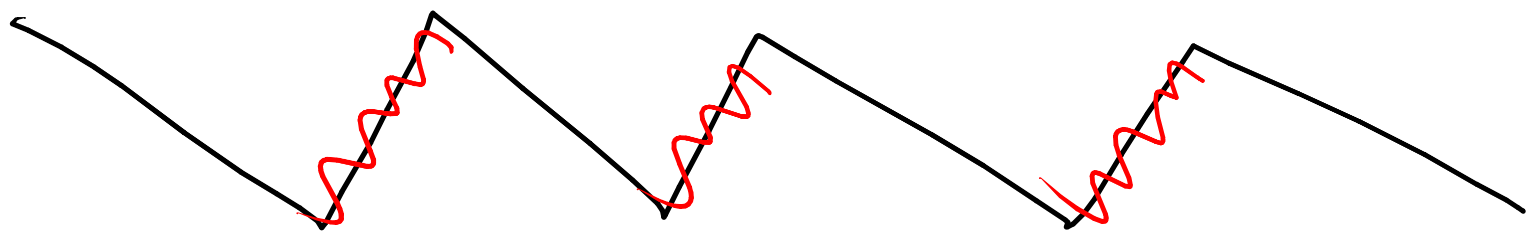
Thm

M is a maximum matching iff there are no M-augmenting paths
(maximum size matching)

Proof

① If $\exists$ an augmenting path then M is not a maximum size


M



Suppose $\exists$ augmenting path.

Suppose edges are $e_0, e_1, e_2, \ldots, e_k$

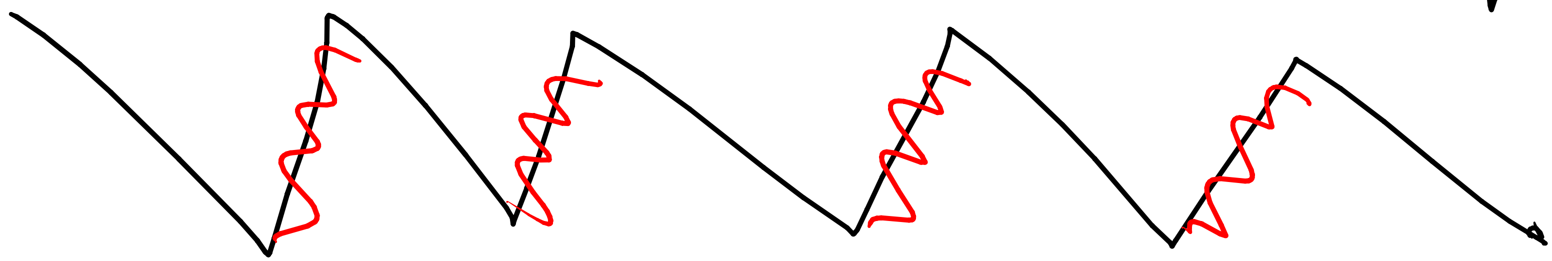
$\in M$ $\notin M$ $\in M$

$$M' = M + e_0 - e_1 + e_2 - e_3 + \cdots + e_k$$

(i) $|M'| = |M| + 1$

(ii) M' is a matching

M' is a matching i.e. $\deg_{M'}(x) \leq 1$ $\forall$ vertices x



Vertex x not on path : $\deg_{M'}(x) = \deg_M(x)$

middle of path $\deg_{M'}(x) = \deg_M(x)$

loses an edge, gains an edge.

endpoints: $\deg_{M'} \to 1$

Suppose now that M is a matching and M' is another matching and

$$|M'| > |M|.$$

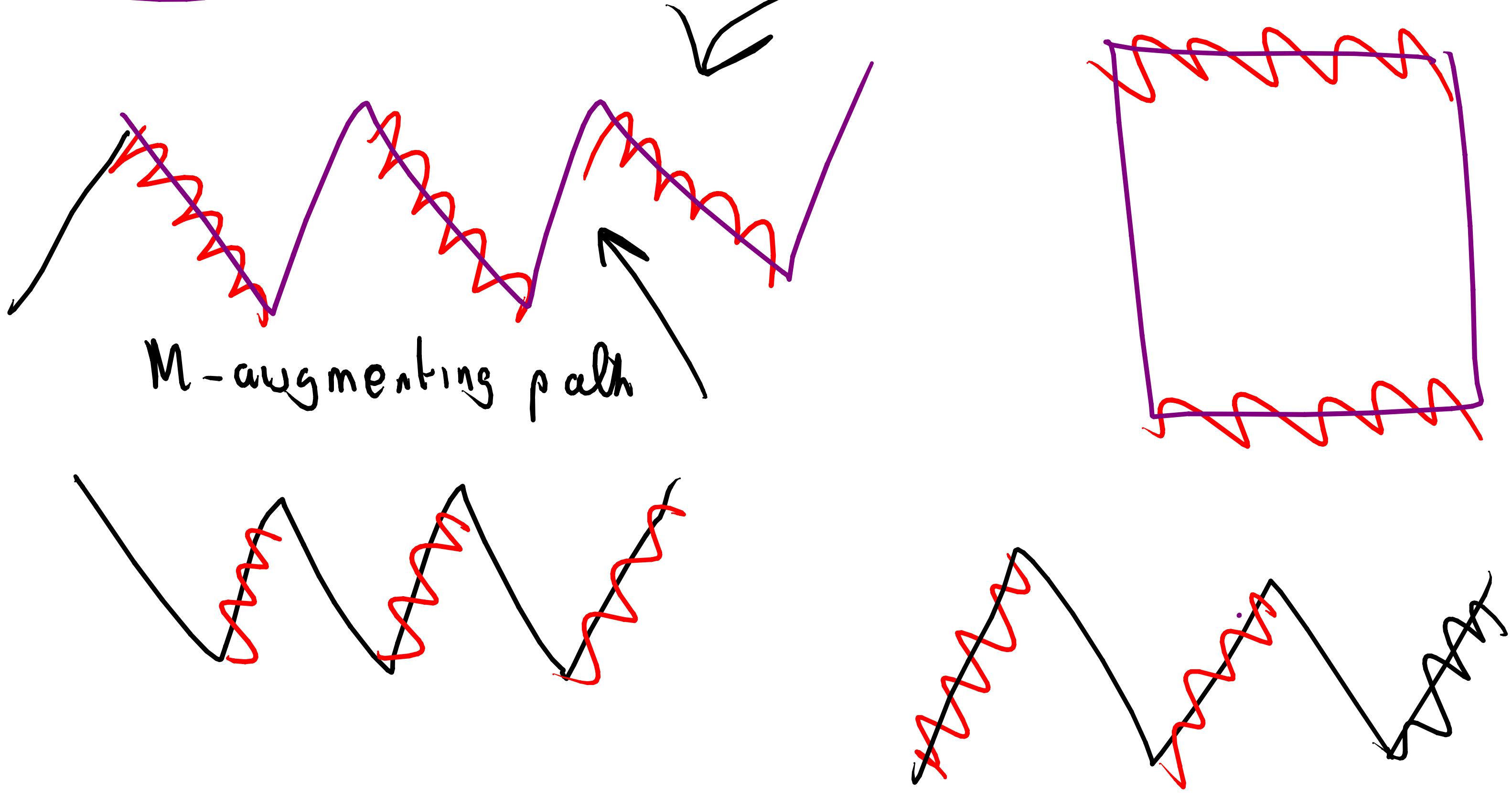
We have to show $\exists$ M-augmenting path

$$M \oplus M' = (M \setminus M') \cup (M' \setminus M)$$

= edges in one of the matchings

$M \oplus M'$

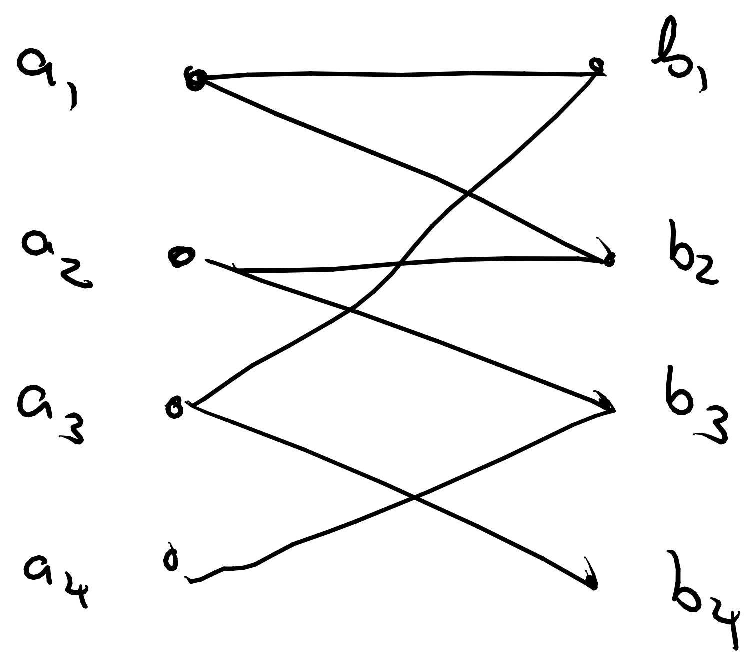
more than



Since $|M'| > |M|$ you must have at least one □

Bipartite Graphs

$G = (A \cup B, E)$



$S \subseteq A$ then

$N(S) = $ nbrs of S

$= \{ b \in B : \exists a \in A$
$\& (a, b) \in E \}$

$N(\{a_1, a_2\}) = \{b_1, b_2, b_3\}$

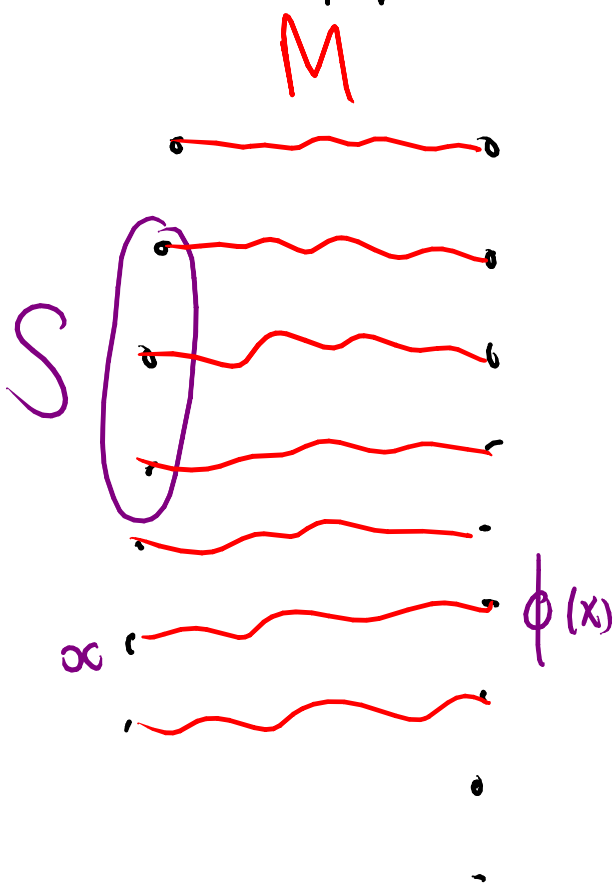
Hall's Theorem

G contains a matching of size $|A|$ iff

$$|N(S)| \geq |S|, \quad \forall S \subseteq A$$

Hall's Condition

<u>Only if</u>: Suppose $\exists$ matching of size $|A|$.



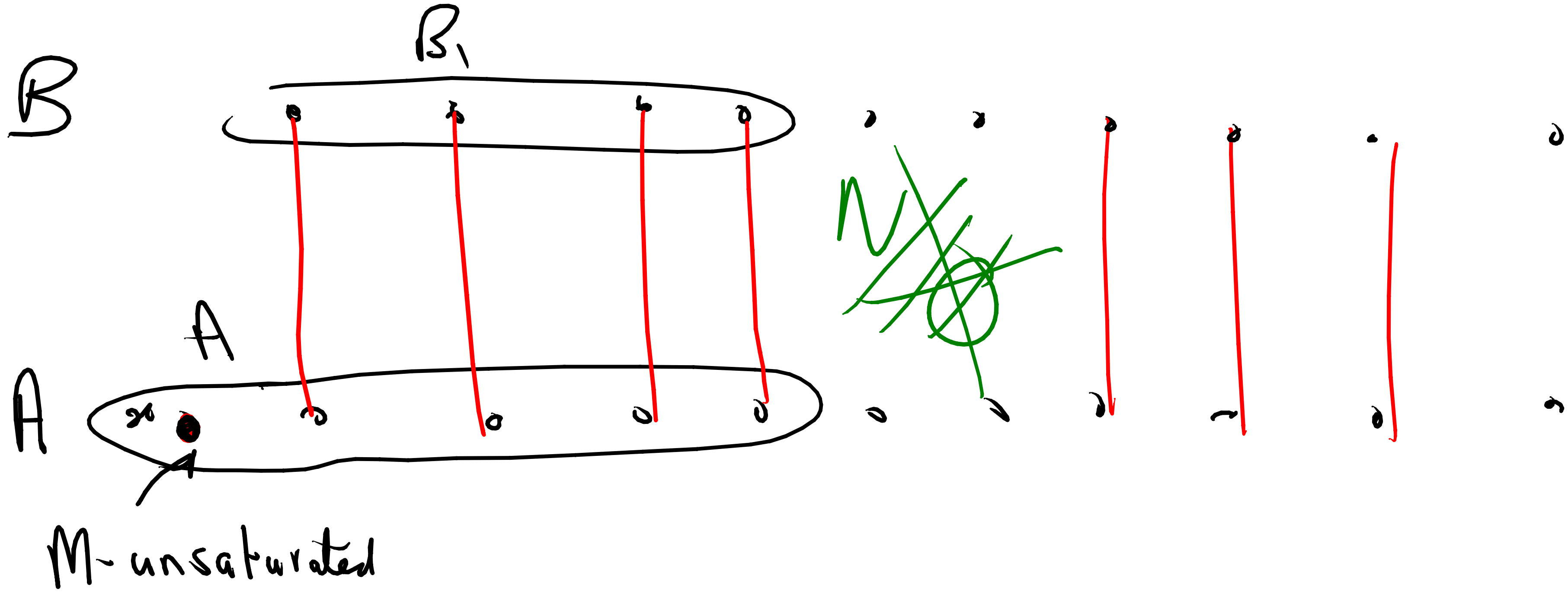
plus extra edges

$x \neq y \Rightarrow \phi(x) \neq \phi(y)$

$$N(S) \supseteq \{\phi(s) : s \in S\}$$

$$| \qquad\qquad | = |S|$$

If: Suppose $\nexists$ matching of size $|A|$



$A_1 = \{ a \in A :$ a is reachable from x by an alternating path $\}$

$B_1 = \{ b \in B :$ . . . . . . $\}$