

21-301 Combinatorics
Homework 9
Due: Monday, December 5

1. There are two boxes. Initially, one box contains m chips and the other contains n chips. Such a position is denoted by (m, n) where $m, n > 0$. A move consists of emptying one of the boxes and dividing the contents of the other between the two boxes with at least one chip in each box. There is a unique terminal position, $(1, 1)$. Find all P positions. Hint: Compute the N, P positions for small m, n and see if you can see the pattern.
2. Consider the following take-away game: In the first move you are not allowed to take the whole pile. After that, if a player removes x chips, then the next player can remove up to $\lfloor 3x/2 \rfloor$ chips. Determine the P positions.
3. Find the set of P -positions for the take-away games with subtraction sets
 - (a) $S = \{1, 3, 5\}$.
 - (b) $S = \{1, 3, 6\}$.

Suppose now that there are two piles and the rules for each pile are as above. Now find the P positions for the two pile game.