21-301 Combinatorics
Homework 8
Due: Monday, November 14

1. Given any sequence of n integers, positive or negative, not necessarily all different, show that some consecutive subsequence has the property that the sum of the members of the subsequence is a multiple of n.

2. Suppose that $a_1, a_2, \ldots, a_n \in [n]$ and $b_1, b_2, \ldots, b_n \in [n]$. An interval I is a set of the form $\{i, i+1, \ldots, j\}$. Let $a_I = a_i + a_{i+1} + \cdots + a_j$ and $b_I = b_i + b_{i+1} + \cdots + b_j$. Show that there exist intervals I, J such that $a_I = b_J$.
   Hint: Assume that $\sum_{i=1}^n a_i \geq \sum_{i=1}^n b_i$. Then show that for each k there exists j and $R_k \in [0, n-1]$ such that $\sum_{s=1}^j a_s = \sum_{t=1}^k b_t + R_k$.

3. Prove that if $n \geq R(2k, 2k)$ and if we 2-color the edges of $K_{n,n}$ then there is a mono-chromatic copy of $K_{k,k}$.