

21-301 Combinatorics
Homework 6
Due: Monday, October 31

1. Let $G = (V, E)$ be an r -regular graph with n vertices i.e. every vertex has degree r . $S \subseteq V$ is a *dominating set* if $w \notin S$ implies that there exists $v \in S$ for which $\{v, w\} \in E$. Show, by the probabilistic method, that G has a dominating set of size at most $\frac{1+\ln r}{r}n$.
2. Let S_1, S_2, \dots, S_m and T_1, T_2, \dots, T_m be two partitions of the set X into sets of size k . Show that there is a set $\{s_1, s_2, \dots, s_m\}$ that is a set of distinct representatives for both S_1, S_2, \dots, S_m and T_1, T_2, \dots, T_m .
3. Let G be a bipartite graph with bipartition X, Y such that the degree $d(x) \geq 1$ for all $x \in X$ and $d(x) \geq d(y)$ for all edges (x, y) of G . Show that G has a matching that covers every vertex of X .
(Hint: Suppose there is no such matching. Consider $S \subseteq X$ with fewer than $|S|$ neighbours and as small as possible.)