

21-301 Combinatorics
Homework 1
Due: Wednesday, September 7

1. How many integral solutions of

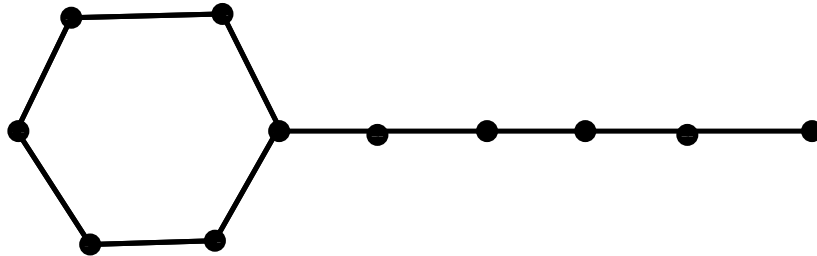
$$x_1 + x_2 + x_3 + x_4 + x_5 = 100$$

satisfy $x_1 \geq 5$, $x_2 \geq 8$, $x_3 \geq -2$, $x_4 \geq 3$ and $x_5 \geq 1$?

2. Show that

$$\sum_{k=0}^n \binom{n}{k} \binom{k}{2} = \binom{n}{2} 2^{n-2}.$$

3. How many ways are there of placing k 1's and $n - k$ 0's at the vertices of the cycle and at the vertices of the path in the diagram below so that each 1 is separated by at least one 0? Thus there will either be $2k$ 1's altogether, when the common vertex has a 0 on it, or $2k - 1$ 1's altogether, when the common vertex has a 1 on it.



The cycle and the path both have n vertices. There are $2n - 1$ vertices altogether.