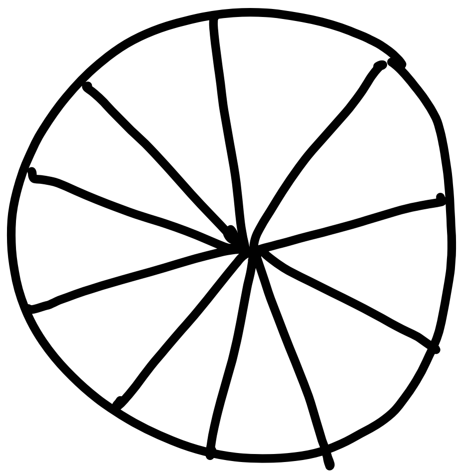
9\30\11

A disc lies in the plane.



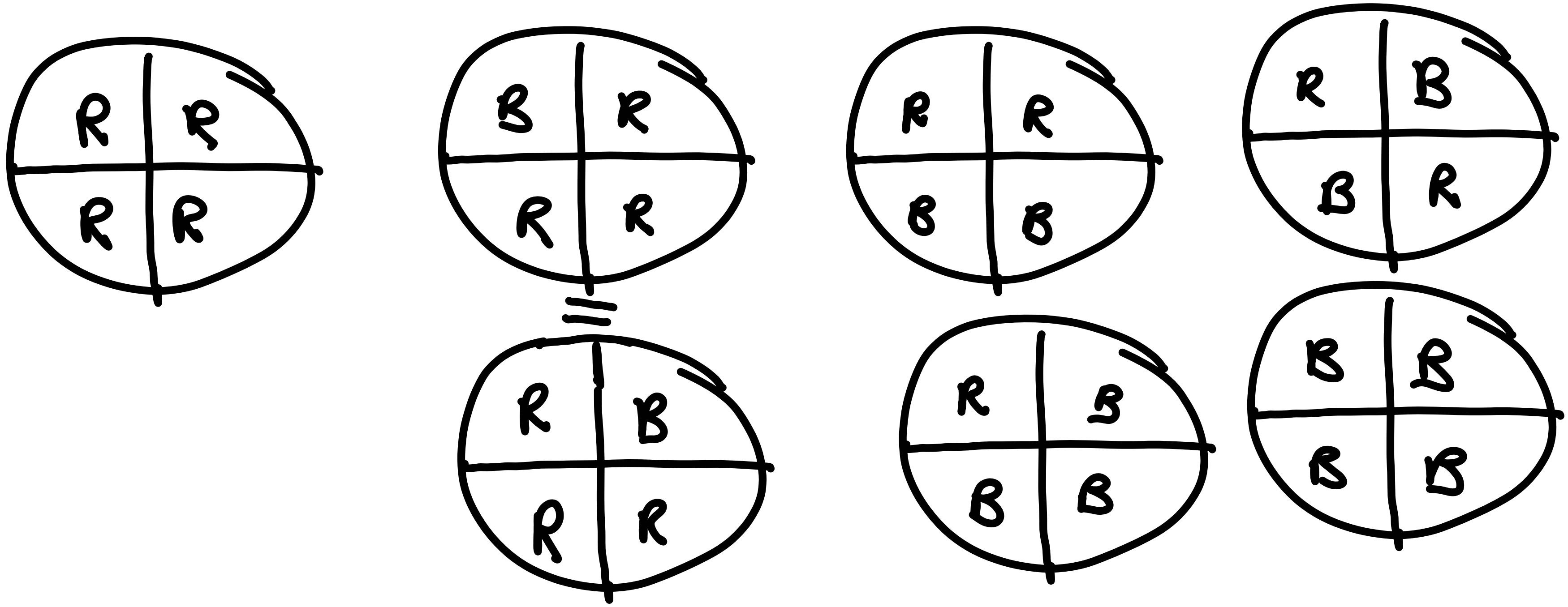
Disc is divided into n sectors.

Each sector can be colored Red or Blue.

How many distinct colorings?

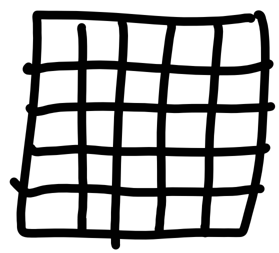
Suppose we can rotate the disc.

One might argue that we only have the following: $n = 4$



Formal Picture

$X = \{$colorings of some object$\}$

$n \times n$ chessboard  $|X| = 2^{n^2}$ if only 2 colors available.

In addition there is a set G of permutations on X.

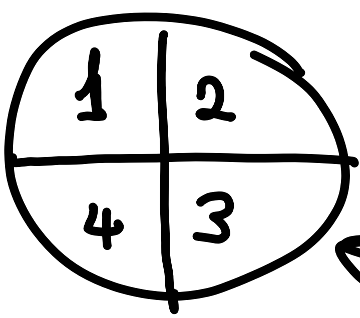
G has a group structure.

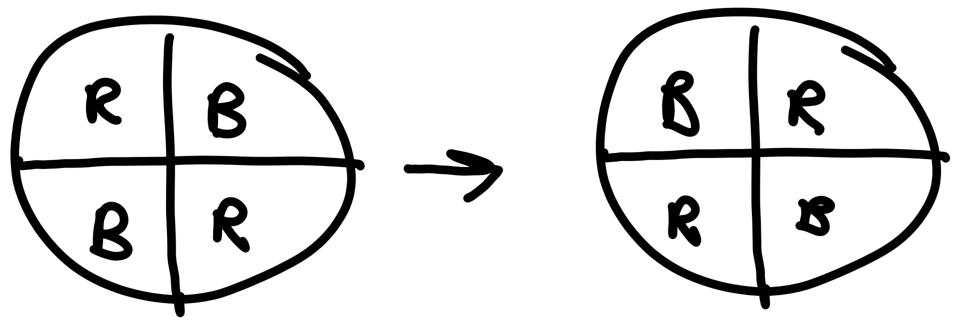
If $g \in G$ and $x \in X$ then $g(x) \in X$

g changes a coloring from x to $g(x)$

g could correspond to a rotation of the object being colored.

Suppose g = rotate 90°





$X = \{$ colorings of  $\}$

$\underline{n=4}$

$G = \{g_0, g_1, g_2, g_3\}$ — 4 rotations.

$G = \{\text{all permutations}\}$ — fewer distinct colorings.
only one

$G = \{$all permutations of the 4 section$\}$ — 5 colorings.

G is a group: $g_1 \circ g_2$ is a permutation.

$$(g_1 \circ g_2)(x) = g_1(g_2(x))$$

g_1: rotate 90° g_2: rotate 180°

$g_1 \circ g_2$ = rotate 270°

(a) $g_1, g_2 \in G \Rightarrow g_1 \circ g_2 \in G$

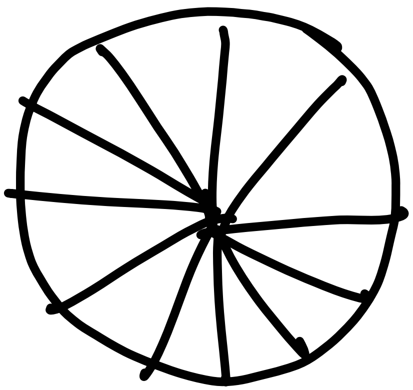
(b) $(g_1 \circ g_2) \circ g_3 = g_1 \circ (g_2 \circ g_3)$

(c) Identity $1_X \in G$

$g \circ 1 = 1 \circ g = g$

(d) $\forall g, \exists g^{-1}$ s.t.

$g \circ g^{-1} = g^{-1} \circ g = 1$



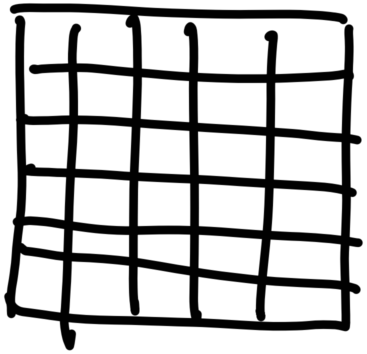
$$D = \{0, 1, 2, \dots, n-1\}$$

$$X = 2^D$$

$$G_{\underline{1}} = \{e_0, e_1, \dots, e_{n-1}\}$$

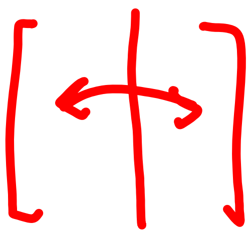
$$e_j = \text{rotate } \frac{2\pi j}{n}$$

$$e_i \circ e_j = e_{i+j}$$

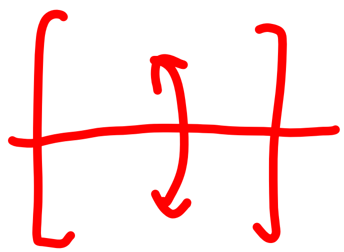


$$X = 2^{[n]^2}$$

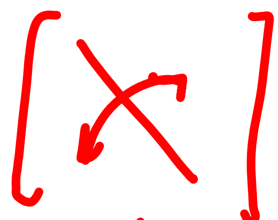
$$G = \{\underbrace{e, a, b, c}_{\substack{\text{rotate} \\ 0, 90^\circ, 180^\circ, 270^\circ}}, \underbrace{p, q}, r, s\}$$



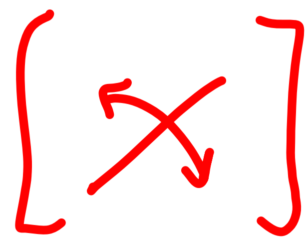
p



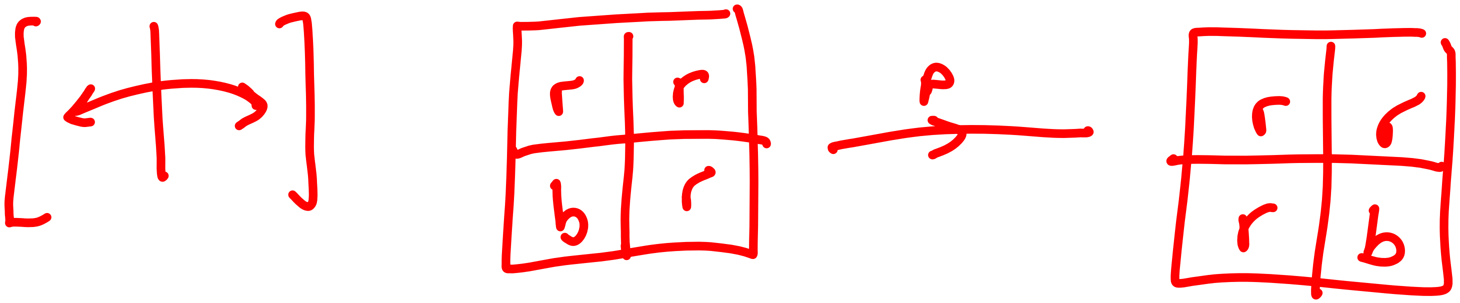
q



r



s

ρ : 

Write $g * x$ in place of $g(x)$: save brackets

If $x \in X$ then its <u>orbit</u>

$$\mathcal{O}_x = \{ y \in X : \exists g \in G \text{ s.t. } g * x = y \}$$

Orbits partition X. Problem. Count # orbits!