9\26\11

Derangements : d_n = # of derangements of $[n]$.

$$n! = \sum_{k=0}^{n} \binom{n}{k} d_{n-k}$$

$$= \sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!} d_{n-k}$$

$$1 = \sum_{k=0}^{n} \frac{1}{k!} \times \frac{d_{n-k}}{(n-k)!}$$

$$a_k \qquad b_{n-k}$$

? $a(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$b(x) = \sum_{n=0}^{\infty} \frac{d_n}{n!} x^n$

$a(x) \times b(x)$

$$1 = \sum_{k=0}^{n} \frac{1}{k!} \times \frac{d_{n-k}}{(n-k)!}$$

Multiply by x^n and sum over n

$$\underbrace{\sum_{n=0}^{\infty} x^n}_{\frac{1}{1-x}} = \underbrace{\sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} \frac{1}{k!} \frac{d_{n-k}}{(n-k)!} \right) x^n}_{a(x)d(x)}$$

$$a(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

$$d(x) = \sum_{n=0}^{\infty} \frac{d_n}{n!} x^n$$

$$\frac{1}{1-x} = e^{x} d(x)$$

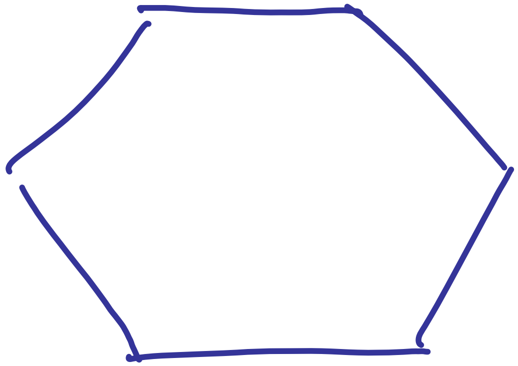
$$a(x) = \frac{1}{1-x}$$

$$b(x) = e^{-x}$$

$$d(x) = \frac{e^{-x}}{1-x}$$

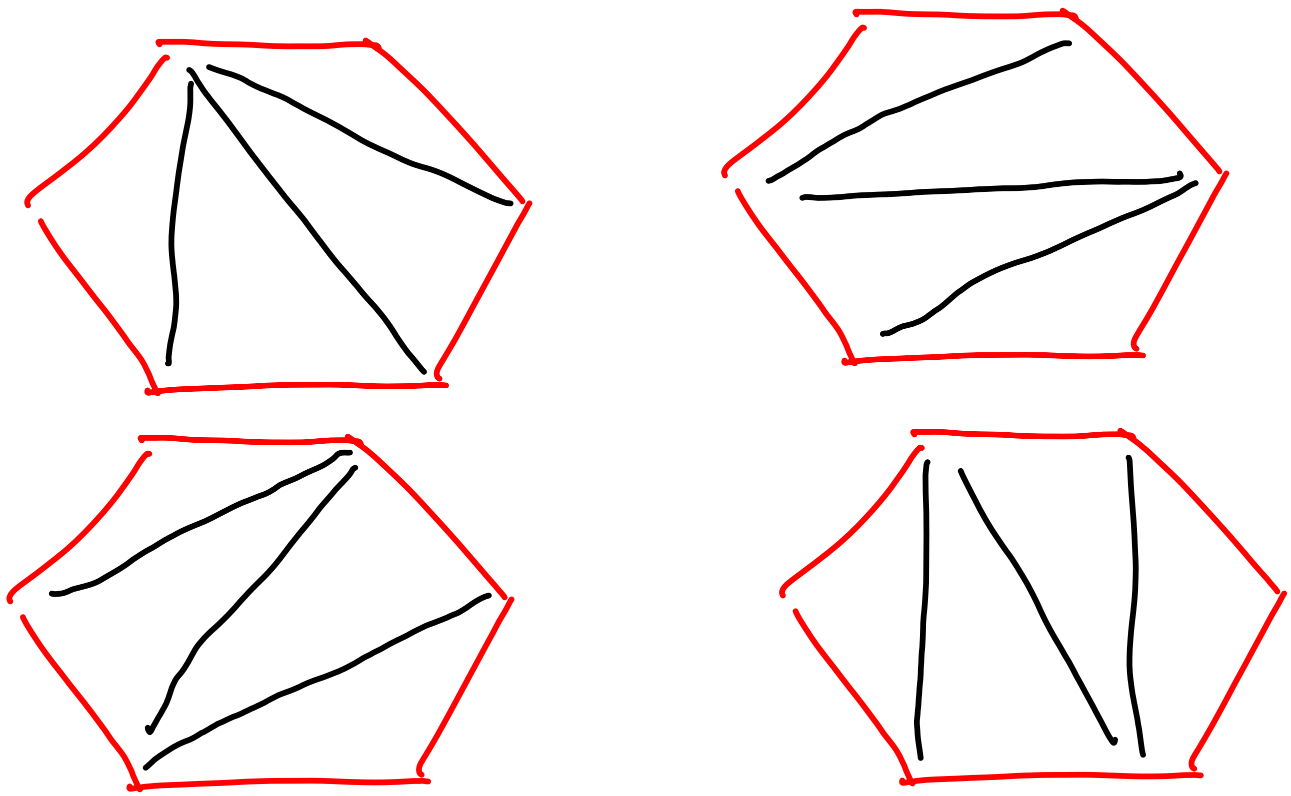
$$= \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} \overset{a_k}{1} \cdot \overset{b_{n-k}}{\frac{(-1)^{n-k}}{(n-k)!}} \right) x^{n}$$

$$\frac{d_n}{n!} = \sum_{k=0}^{n} \frac{(-1)^{n-k}}{(n-k)!} = \sum_{k=0}^{n} \frac{(-1)^{k}}{k!}$$



a_n = # of ways of triangulating P_{n+1}.

$$= \sum_{k=0}^{n} a_k \, a_{n-k} \qquad n \geq 2$$



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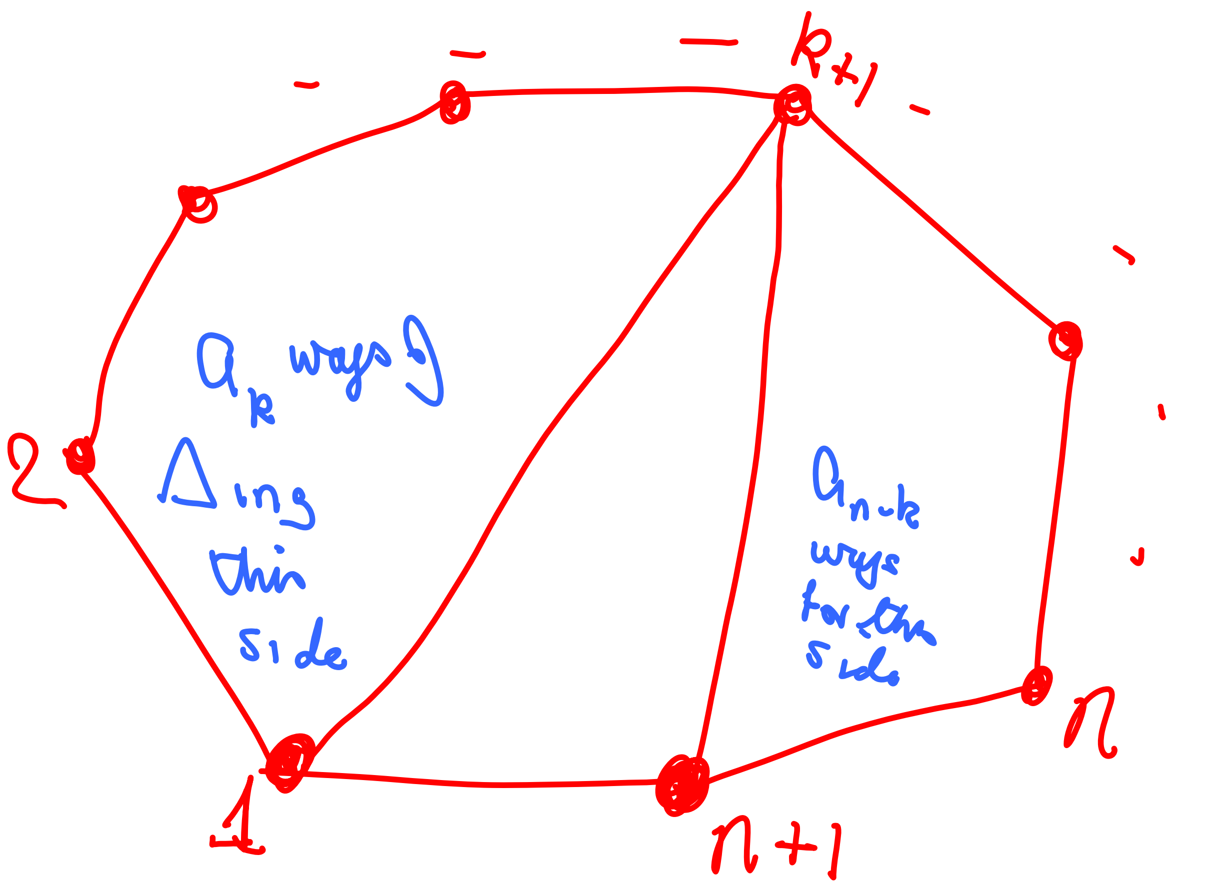
$$a_n = \sum_{k=0}^{n} a_k a_{n-k} \qquad n \geq 2$$

$$a_0 = 0, \quad a_1 = a_2 = 1$$



k+1 goes from 2 to n

k goes 1 to n-1

$$a_n = \sum_{k=0}^{n} a_k a_{n-k} \qquad n \geq 2$$

$$a_0 = 0, \quad a_1 = a_2 = 1$$

Multiply by x^n and sum over $n \geq 2$

$$\underbrace{\sum_{n=2}^{\infty} a_n x^n}_{a(x) - a_0 - a_1 x} = \sum_{n=2}^{\infty} \left(\sum_{k=0}^{n} a_k a_{n-k} \right) x^n$$

$$= a(x) - x = a(x)^2$$

$$\sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} a_k a_{n-k} \right) x^n = a(x)^2$$

$$a(x) - x = a(x)^2$$

Solve quadratic for a-unknown

$$a(x) = \cancel{\frac{1+\sqrt{1-4x}}{2}} \quad \text{or} \quad \frac{1-\sqrt{1-4x}}{2}$$

$\neq 0$ when $x = 0$

$$a(x) = \frac{1-(1-4x)^{1/2}}{2}$$