11\7\11

Schur's Theorem

$$r_k = R(\underbrace{3, 3, \cdots, 3}_{k}; 2)$$

Minimum n such that if you k-color the edges of K_n then there must be a monochromatic triangle

Theorem

If $S_1, S_2, \cdots, S_k$ is a partition of $\{1, 2, \cdots, r_k\}$ then $\exists i$ and $x, y, z \in S_i$ such that $x + y = z$.

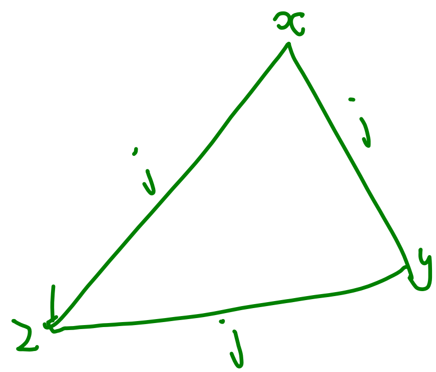
Given $S_1, S_2, \dots, S_k$ and $n = r_k(3, \dots, 3; 2)$

we define a k-coloring of the edges of K_n.

If (u,v) is an edge of K_n and $|u-v| \in S_j$

then we color (u,v) with j.

$\exists$ a mono-chromatic triangle



$x > y > z$

$a = x - y \in S_j$ $\quad a + b = c$

$b = y - z \in S_j$

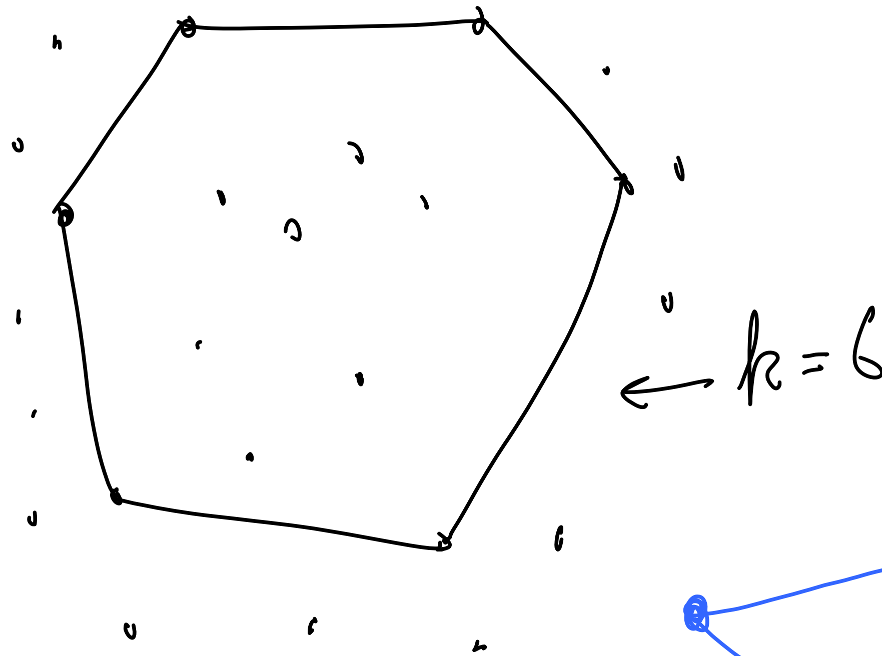
$c = x - z \in S_j$

A set of n points X in the plane is in general position if no 3 points are collinear.

If we 2-color all the 3-tuples in $[n]$ then $\exists$ a k-set all of whose 3-subsets have same color

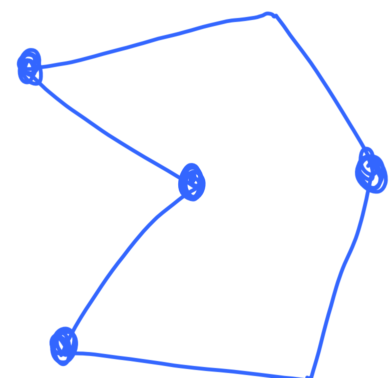
Theorem

If $n \geq R(k,k;3)$ then X contains a set of points Y, $|Y| = k$, such that Y is the set of vertices of a convex polygon. (Happy Ending Theorem)



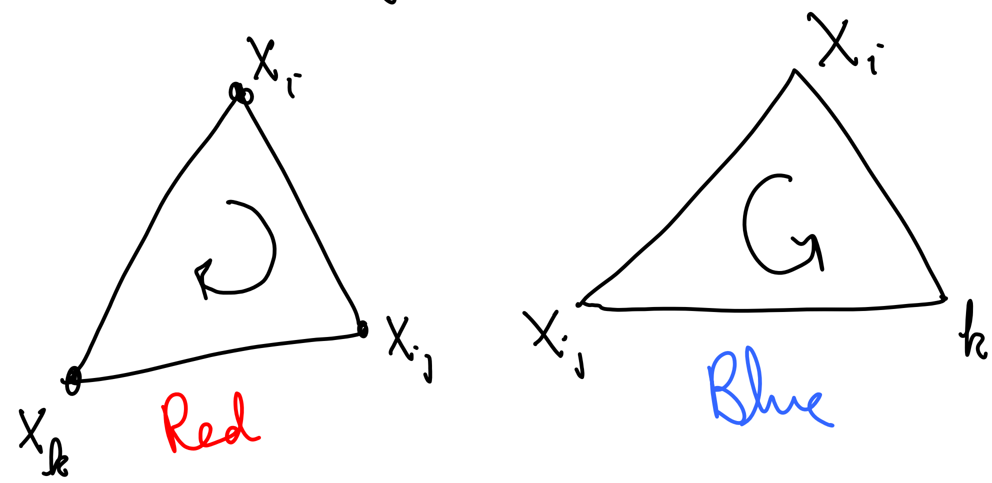
Observation

If every 4-subset of Y is convex then Y is convex.



So we must find a k-set all of whose 4 sets are convex.

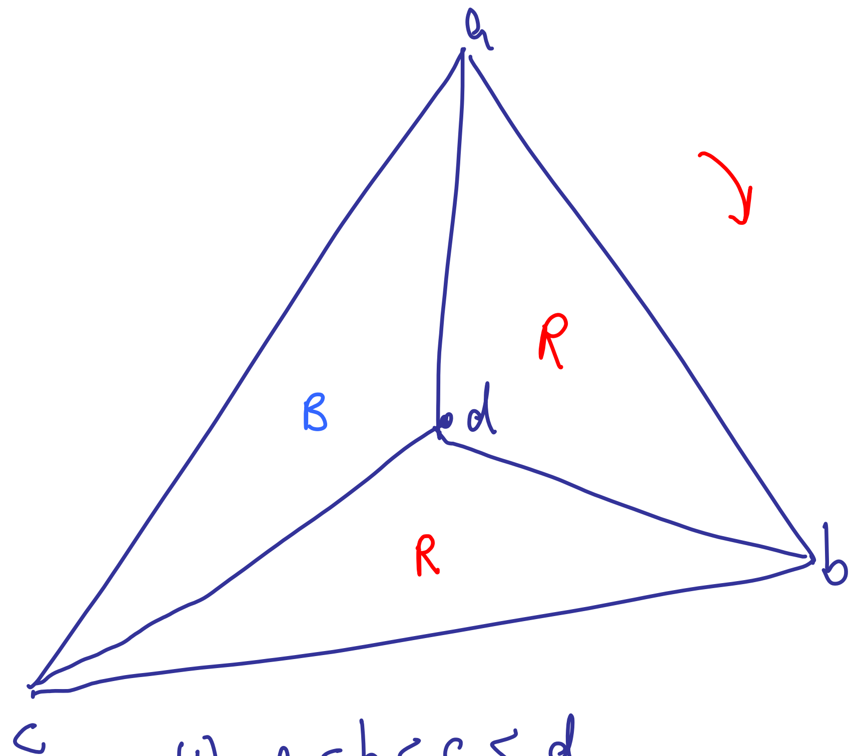
To apply "Ramsey" we must color the triples.



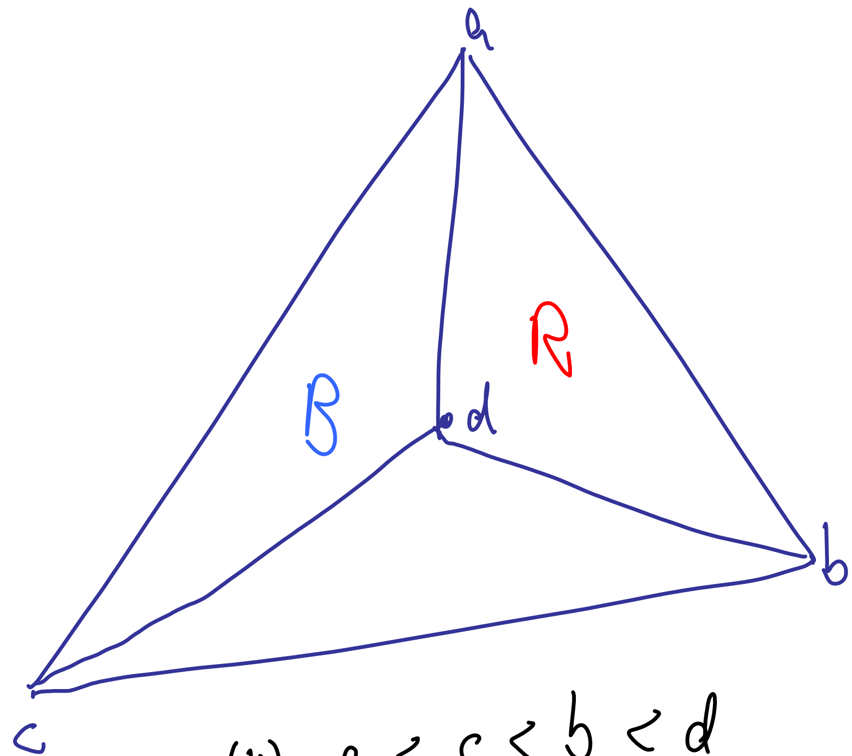
$i < j < k$

Each 3-tuple is given an orientation.

Ramsey says that $\exists$ a k-tuple all of whose triples have same color. $\Rightarrow \triangle \nexists$



(1) $a < b < c < d$



(11) $a < c < b < d$