11\30\11

One pile game.

If a player removes x then next player removes $y \leq f(x)$.

f is a monotone increasing function and $f(x) \geq x$, for all x.

P positions: $H_1, H_2, \dots\dots$

$H_1 = 1$; $H_{j+1} = H_j + H_l$ $H_l = \min_{i \leq j} \{ H_i : f(H_i) \geq H_j \}$

l exists, since $f(H_j) \geq H_j$

We use

Thm

Every positive integer n can be *uniquely* written as

$$n = H_{j_1} + H_{j_2} + \cdots + H_{j_p}$$

where $f(H_{j_i}) < H_{j_{i+1}}$ for all i.

So every integer has a H-binary expansion

Put a 1 in position j_i for each i

Winning strategy, for $p \geq 2$, is to remove H_{j_p} tokens.

After this, next player cannot reduce number of bits in the H-expansion.

If $f(x) = 2x$ then $H = \{1, 2, 3, 5, 8, \dots\}$

$= \{F_1, F_2, F_3, \dots$

Proof by induction

$$H_{j+1} = F_j + \min_{i \le j} \{ F_i : 2F_i \geq F_j \}$$

$$= F_j + F_{j-1}$$

because $2F_{j-2} < F_j$

Geography

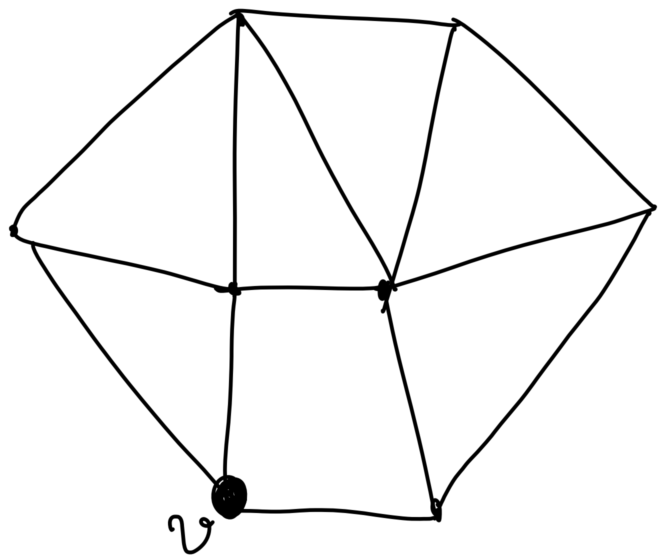
4 sorts

	undirected	directed
vertex	P	Pspace Hard
edge	Pspace Hard	Pspace Hard

Move deletes vertex or edge

Graph is directed or undirected

Undirected Vertex Geography



G

Claim: this is a winning position iff every maximum matching in G covers v.

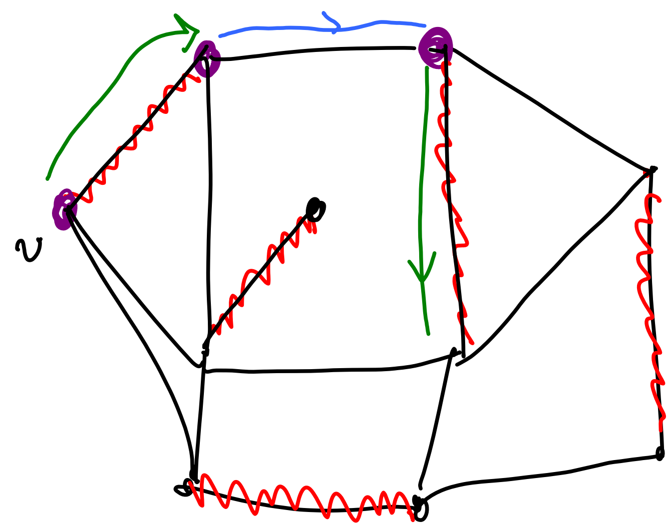
Initial position: start at v.

Suppose v is in all maximum matchings.

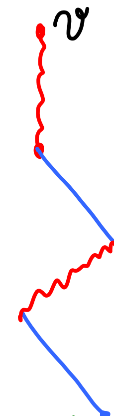
Let M be a maximum matching

First player, always follows M.

Game



How can First player lose?

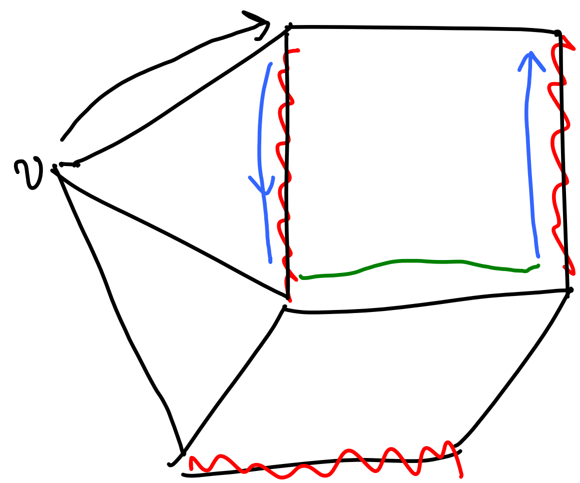


No matching edge

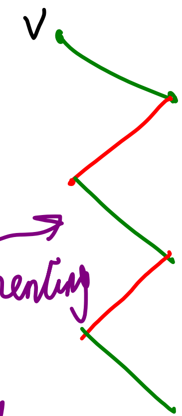
There is a maximum matching that does not cover v.

Now suppose that there is a maximum matching M that does not cover v.

Second player follows M.



How can second player lose?



augmenting

$\Downarrow$

M not maximum.

No matching edge.