11/2/11

$a_1, a_2, \dots, a_{k^2+1}$ contains a monotone sequence of length $k+1$.

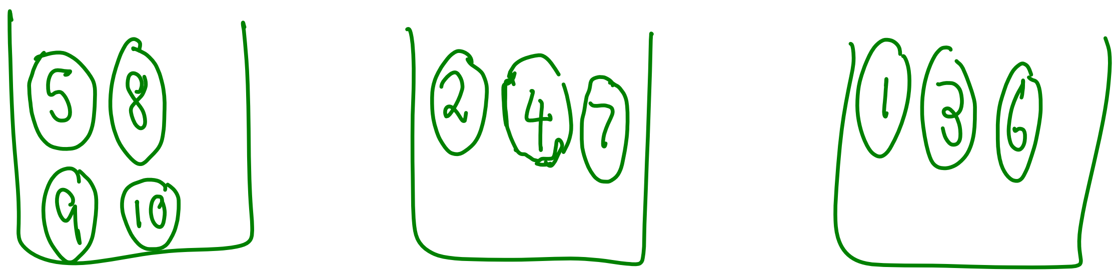
Let $a_i \le a_i^{(1)} \le a_i^{(2)}, \dots \le a_i^{(l)}$ be the longest monotone increasing sequence that starts at a_i. Let $l(i)$ be its length.

3 6 2 5 8 2 4 9 5

$$l(4) = 3$$

$a_4 = 5$, longest sequence starting at a_4 = 5, 8, 9

3 6 2 5 8 1 4 7 5 0



$a_5 \geq a_8 \geq a_9 \geq a_{10}$

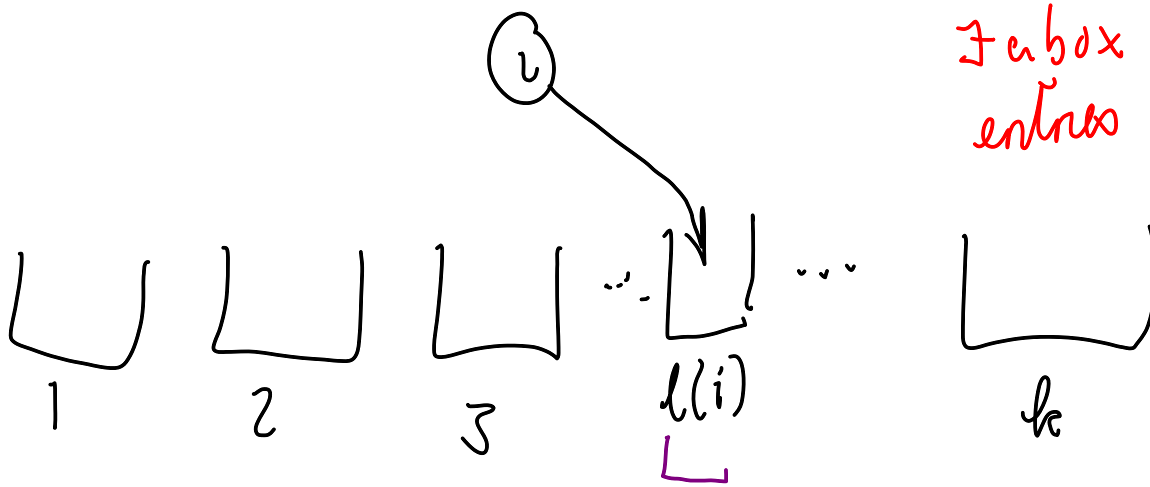
Case 1: $\exists i : \ell(i) \geq k+1$

Here there exists a monotone increasing sequence of length $k+1$.

Case 2: $\ell(i) \leq k, \ \forall i$

$\ell(1), \ell(2), \ldots, \ell(k^2+1)$ consists of k^2+1 numbers between 1 and k.

$\exists$ a box with $\geq k+1$ entries



There exists an L and $i_1 < i_2 < i_3 < \dots < i_{k+1}$

such that $\ell(i_t) = L$, $1 \leq t \leq k+1$.

This sequence $a_{i_1}, a_{i_2}, \dots a_{i_{k+1}}$ is monotone decreasing

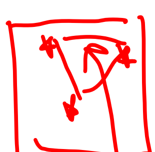
Suppose

$$a_{i_t} < a_{i_{t+1}} \leq a^{(1)}_{i_{t+1}} \leq \dots \leq a^{(L)}_{i_{t+1}}$$

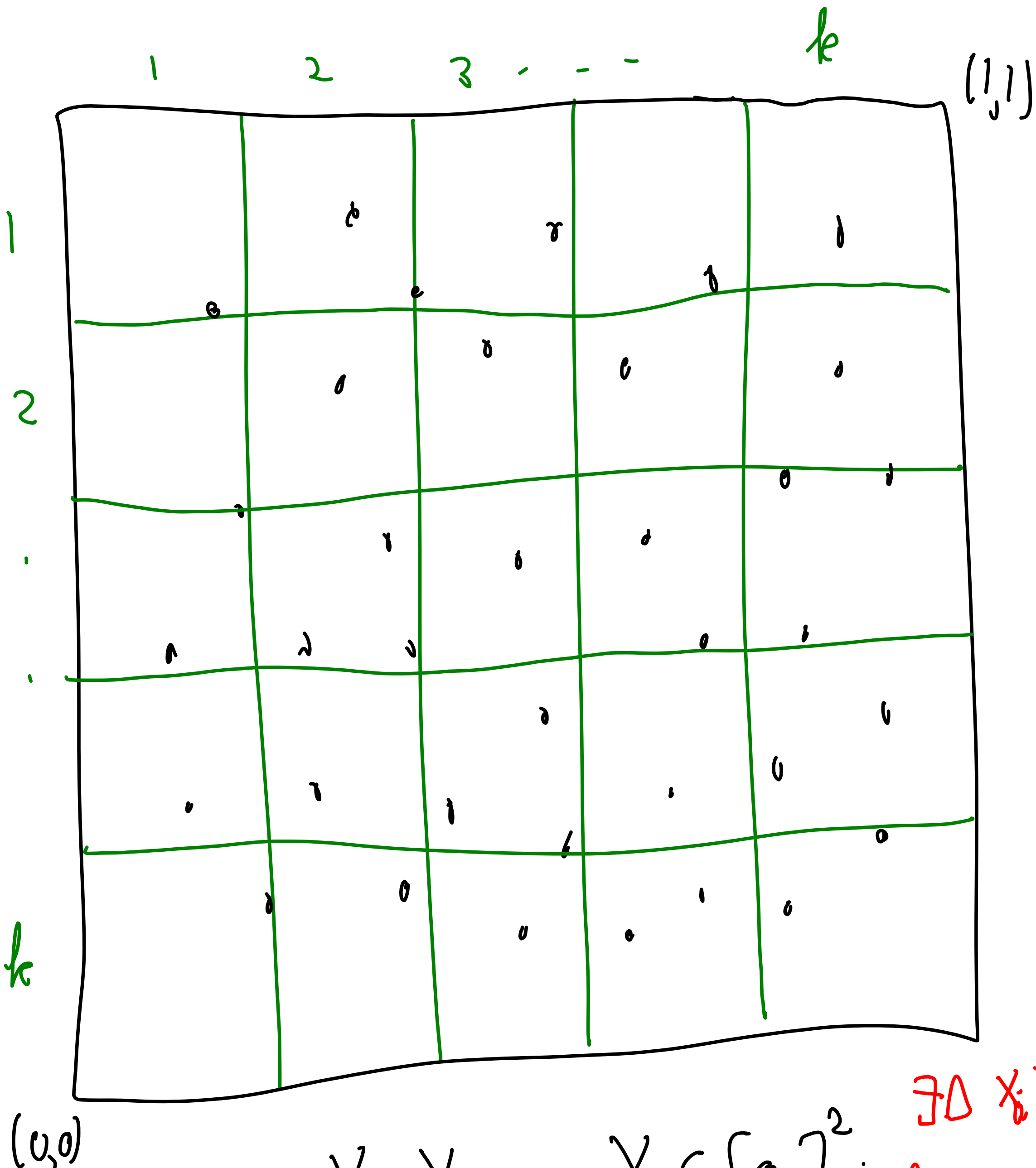
This says that $\ell(i_t) \geq L+1$, contradiction.

Choose largest k so that we know there is a square with 3 points

$k^2 < \frac{n}{2}$

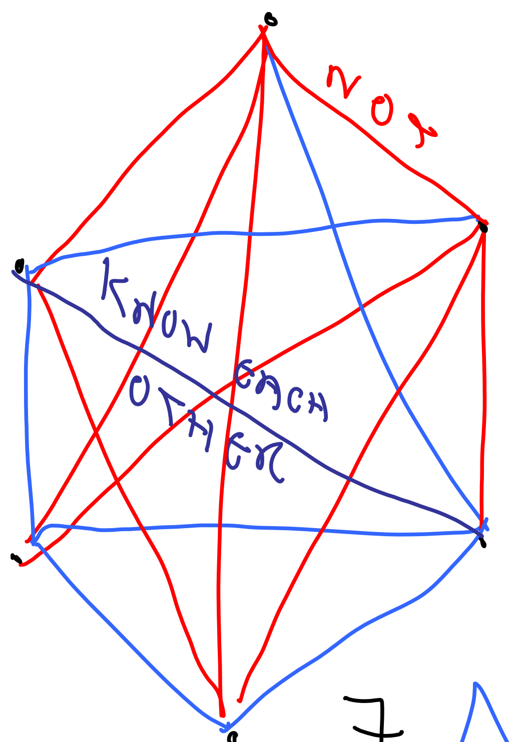


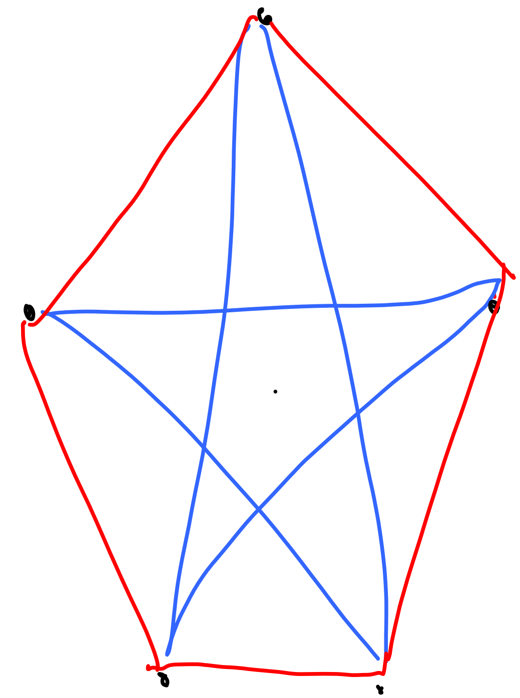
Area $\leq \frac{1}{2} \times \frac{1}{k^2}$



$X_1, X_2, \dots, X_n \in [0,1]^2$: $\exists \Delta\ X_i, X_j, X_k$ of area $\leq \frac{2}{n}$

Ramsey Theory (After Frank Ramsey)





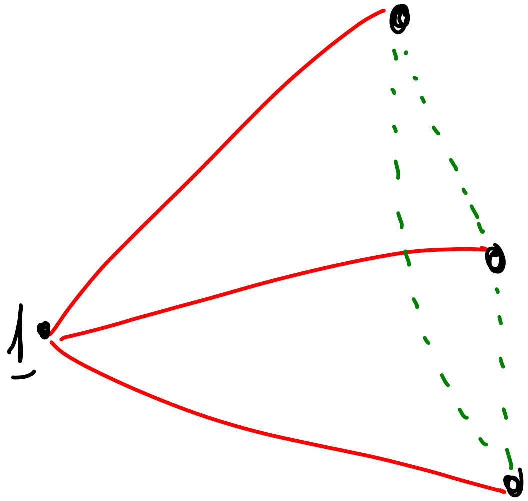
6 people $\exists$ $\triangle$ or $\triangle$ or both

$R(3,3) = 6$

K_6

all blue

$\geq$ one red



5 edges incident with 1.

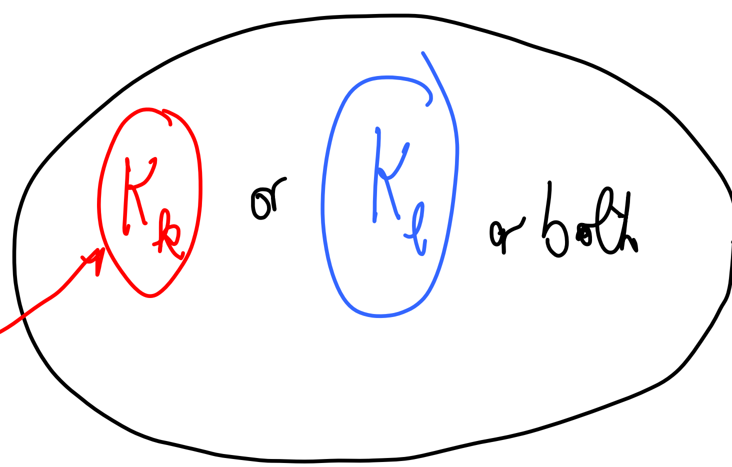
≥ 3 must have same color

PHP

Ramsey's Theorem

For all pairs of positive integers k, ℓ $\exists$ an integer $R(k,\ell)$ such that if $n \geq R(k,\ell)$ then in any Red/Blue coloring of the edges of K_n

a set of k vertices with all $\binom{k}{2}$ pairs colored Red



K_n

$R(3,3) = 6$

$R(4,4) = 18$

$R(5,5) = ?$