10/5/11

$$\#\text{orbits} = \frac{1}{|G|} \sum_{x \in X} |S_x|$$

#1's in matrix, summed by row

$$\begin{array}{cc} & g \rightarrow \\ x \downarrow & \begin{bmatrix} \mathbb{1}_{g * x = x} \\ 0 \text{ otherwise} \end{bmatrix} \end{array}$$

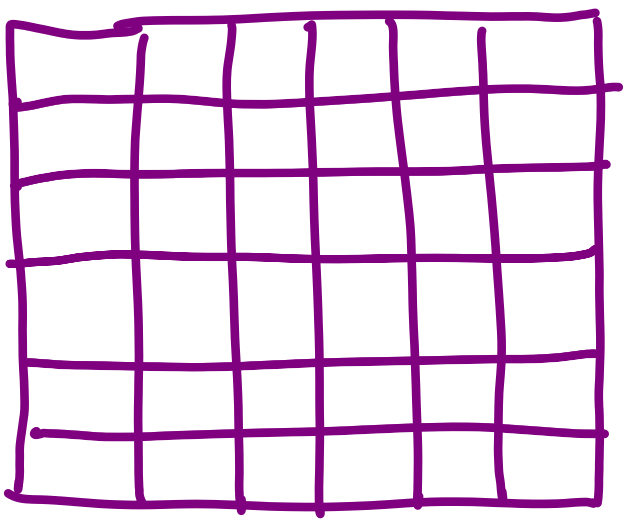
← # in row x =

$\# = |\{x : g * x = x\}| = |\mathrm{Fix}(g)|$

So,

$$\#\text{ of Orbits} = \frac{1}{|G|} \sum_{g \in G} |\mathrm{Fix}(g)|$$

$n=6$: Chessboard



2 colors

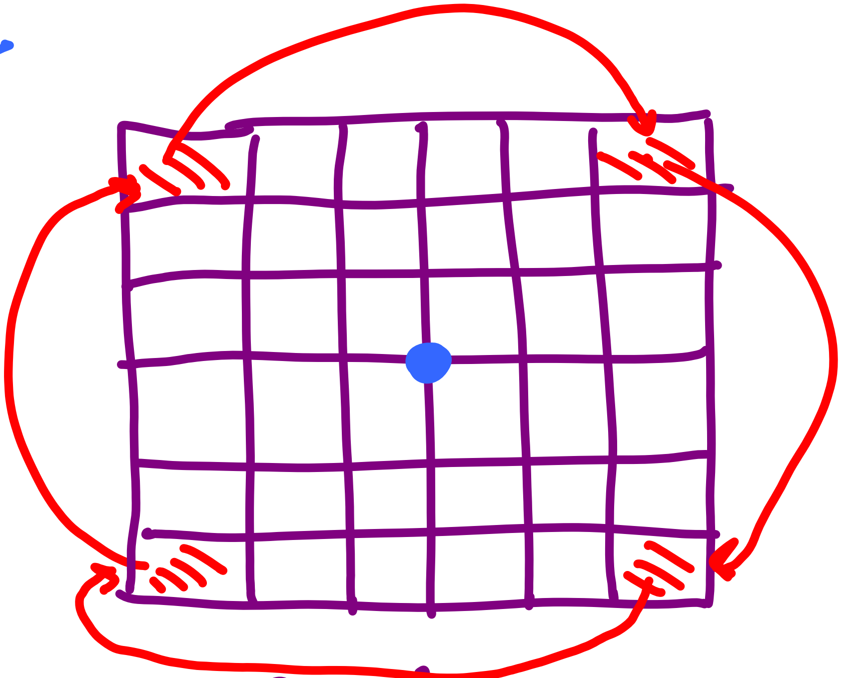
$G = \{e, a, b, c, p, q, r, s\}$

$$\# = \frac{1}{8}\left(\underset{e}{2^{36}} + \underset{a}{2^{9}} + \underset{b}{2^{18}} + \underset{c}{2^{9}} + \underset{p}{2^{18}} + \underset{q}{2^{18}} + \underset{r}{2^{21}} + \underset{s}{2^{21}} \right)$$

This can be carried out for any n.

n odd & n even are slightly different.

a



2 colors

cycles of the permutation

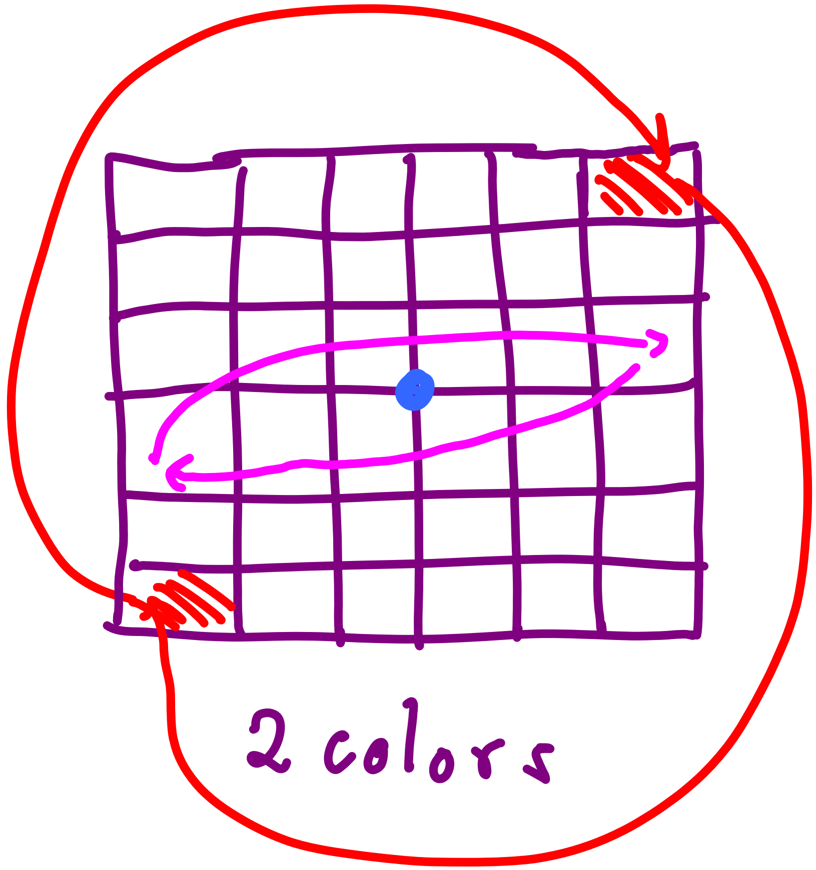
a rotates by $90°$

These 4 squares have to be the same color if $x \in \mathrm{Fix}(a)$

True for all cycles

9 cycles in all.

b

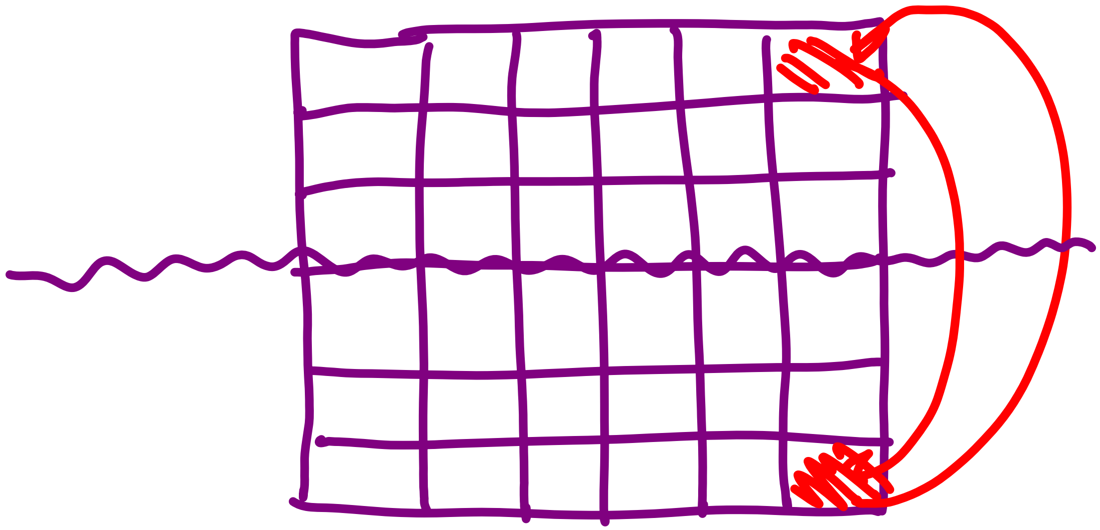


2 colors

b rotation 180°

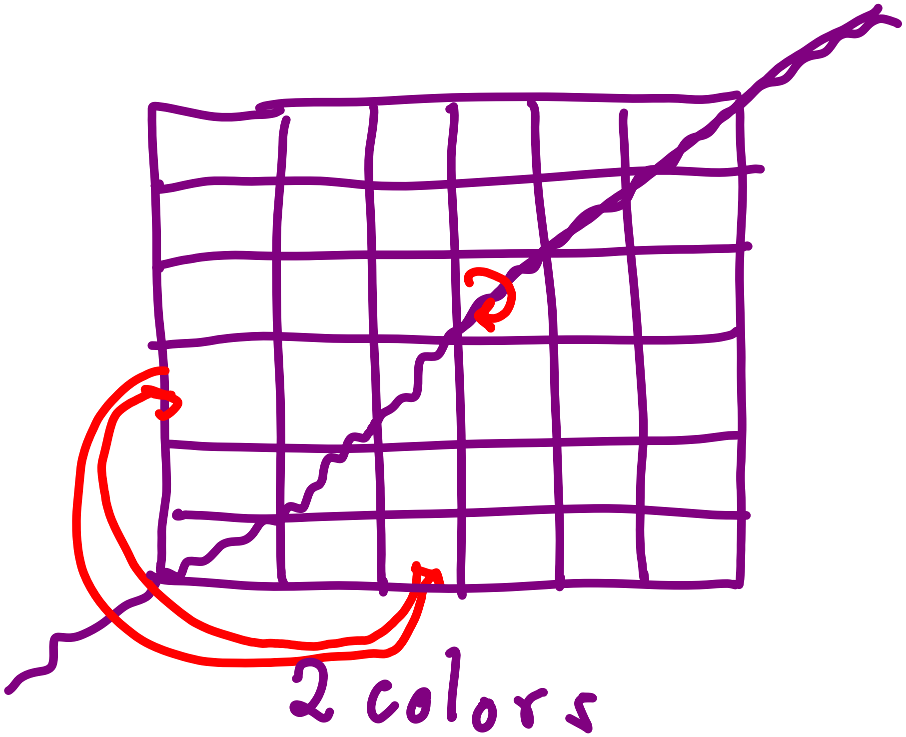
Now there are 18 cycles.

p



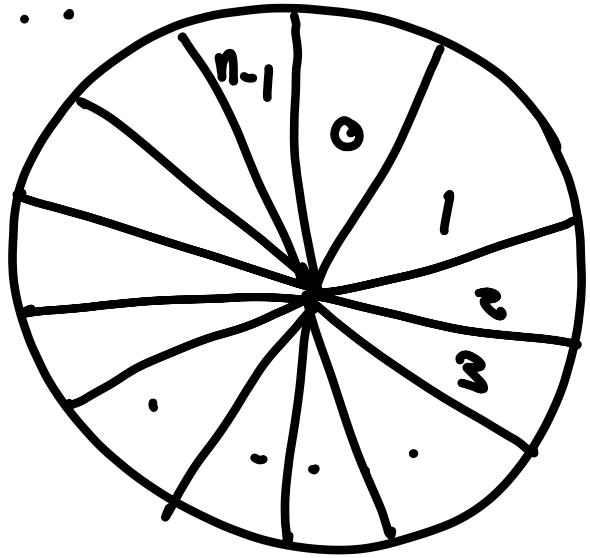
2 colors

18 cycles



$$\# \text{ cycles} = 21 = \frac{6 \times 7}{2}$$

Example 1



n sectors

2 colors

$$G = \{e_0, e_1, e_2, \dots, e_m, \dots, e_{n-1}\}$$

e_m: rotate by $\frac{2\pi}{n} \times m$

$$\#\text{orbits} = \frac{1}{n} \sum_{m=0}^{n-1} 2^{a_m}$$

$a_m = \#$ cycles in e_m

$= \text{g.c.d.}(m, n)$

Cycles of $\mathcal{C}_m$: rotate m sectors

$$0 \longrightarrow m \longrightarrow 2m \longrightarrow 3m \longrightarrow \cdots\cdots \longrightarrow am$$

first a for which am is a multiple of n.

$$am = \text{l.c.m}(m, n)$$

least common multiple

$$= \frac{mn}{\text{g.c.d.}(m,n)} \leftarrow d$$

greatest common divisor.

$$a = n/d$$

cycles is d.