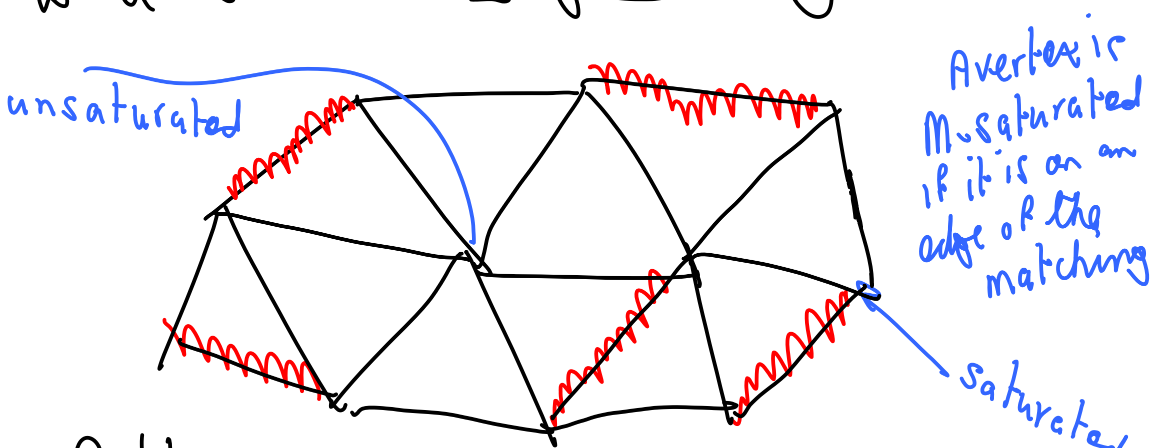
10/24/11

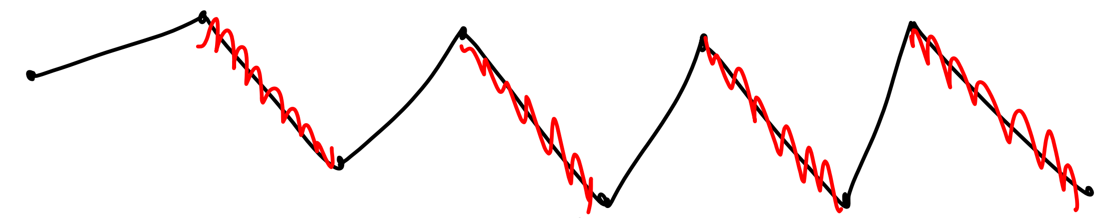
Matchings

A matching M in a graph $G=(V,E)$ is a set of disjoint edges.

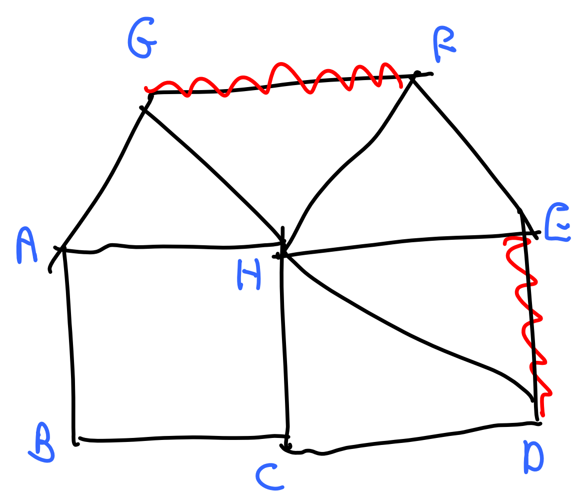


Problem: find maximum size matching

M - alternating path



Edges are alternately in M and not in M.



DEFG ALTERNATING

CDEFG "

CDEFGA "

AUGMENTING

Starts and ends at an unsaturated vertex.

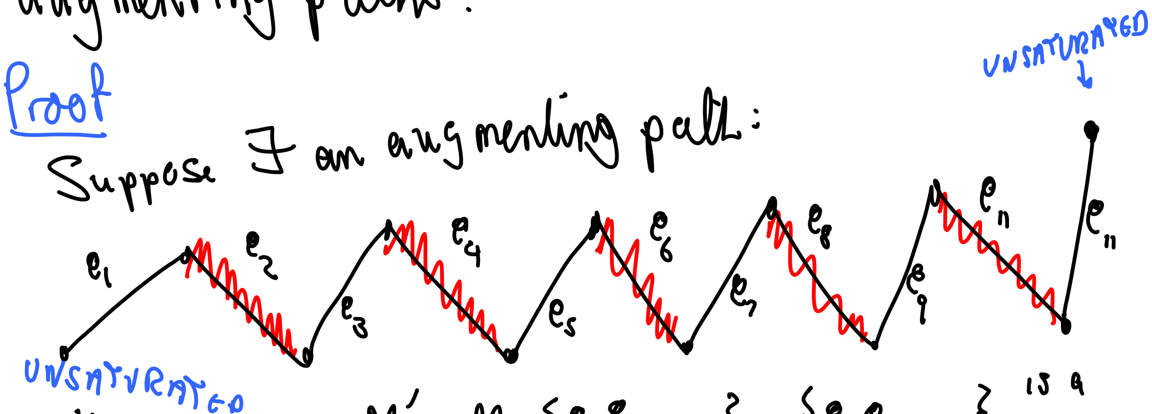
M is a <u>maximum</u> matching if no matching M' has more edges.

<u>Theorem</u>

M is a maximum matching iff it has no augmenting paths.

Proof

Suppose $\exists$ an augmenting path:



$|M'| = |M| + 1$ $\quad M' = M + \{e_1, e_3, \dots\} - \{e_2, e_4, \dots\}$ is a matching

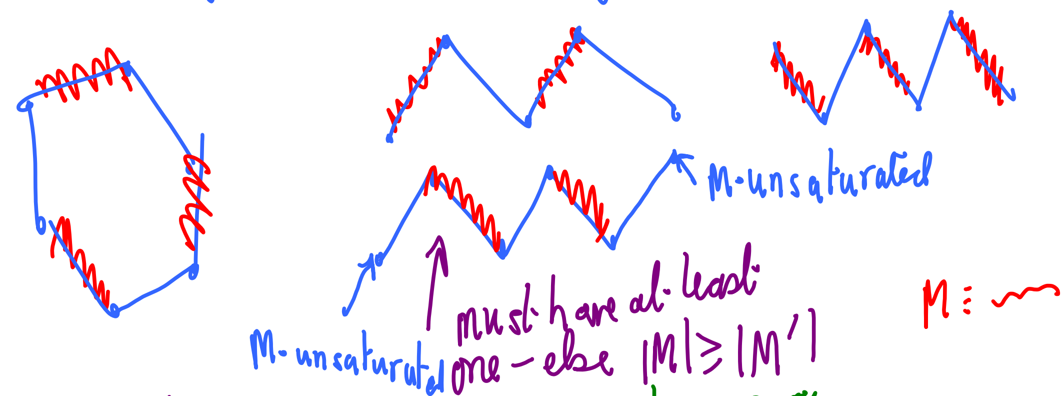
Suppose now that $|M'| > |M|$ is another matching.

Consider $$M' \oplus M = (M / M') \cup (M' \setminus M)$$

edges in one of M, M'.

Components of $\;:\;$ Vertices have degree ≤ 2 in $M \oplus M'$.

A graph with maximum degree ≤ 2 is a collection of vertex disjoint paths and cycles.

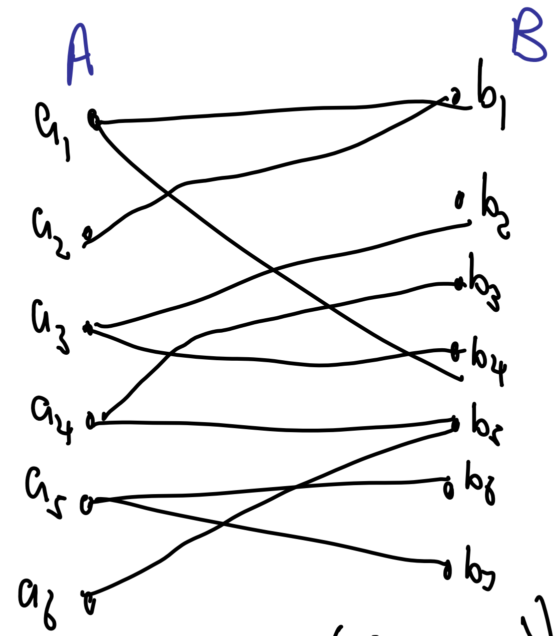


must have at least one – else $|M| \geq |M'|$

M-augmenting path.

Bipartite Graphs

When does $\exists M$ s.t. $|M| = |A|$?



$N(S) := \{ v \notin S : \exists w \in S \text{ s.t. } (w,v) \in E \}$

$N(\{a_2, a_3\}) = \{b_1, b_2, b_4\}$

$|M| \leq \min\{|A|, |B|\}$ for any matching M.

Systems of distinct representatives

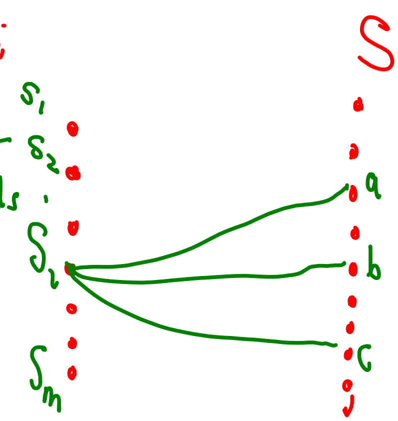
$S_1, S_2, \dots, S_m$ are arbitrary sets.

$\underbrace{s_1, s_2, \dots, s_m}_{\text{distinct}}$ is an SDR if

$$s_1 \in S_1$$
$$s_2 \in S_2$$
$$\vdots$$
$$s_m \in S_m$$

$S = \bigcup S_i$

each vertex corresponds to a set S_i



$S_i = \{a, b, c\}$

$\exists$ an SDR iff $\exists$ matching of size m.

(S_1, s_1)
(S_2, s_2) $\dots$ (S_m, s_m)