21-301 Combinatorics
Homework 9
Due: Friday, November 5

1. Given any sequence of n integers, positive or negative, not necessarily all different, show that some consecutive subsequence has the property that the sum of the members of the subsequence is a multiple of n.

2. Prove that if n is odd then for any permutation π of the set $\{1, 2, ..., n\}$ the product $P(\pi) = (1 - \pi(1))(2 - \pi(2))...(n - \pi(n))$ is necessarily even.

3. Let G_1, G_2 be fixed graphs. Let $r(G_1, G_2)$ be the smallest integer$N$ such that if we two-color the edges of the complete graph K_N there is a Red copy of G_1 or a Blue copy of G_2, or both. Show that if P_3 is a path of length 3 and C_4 is a 4-cycle then $r(P_3, C_4) = 5$. (A triangle is not to be considered to be a path of length 3).