21-301 Combinatorics
Homework 8
Due: Monday, November 1

1. Let $\mathcal{A}$ be an intersecting family of subsets of $[n]$ such that $A \in \mathcal{A}$ implies $k \leq |A| \leq \ell \leq n/2$. Show that
$$|\mathcal{A}| \leq \sum_{i=k}^{\ell} \binom{n-1}{i-1}.$$

2. Let $m = \lfloor n/2 \rfloor$. Describe a family $\mathcal{A}$ of size $2^{n-1} + \binom{n-1}{m-1}$ that has the following property: If $A_1, A_2 \in \mathcal{A}$ are disjoint then $A_1 \cup A_2 = [n]$.

3. Subsets $A_i, B_i \subseteq [n]$, $i = 1, 2, \ldots, m$ satisfy $A_i \cap B_i = \emptyset$ for all i and $A_i \cap B_j \neq \emptyset$ for all $i \neq j$. Show that
$$\sum_{i=1}^{m} \frac{1}{\binom{|A_i|+|B_i|}{|A_i|}} \leq 1.$$