21-301 Combinatorics
Homework 3
Due: Monday, September 13

1. Suppose that in the Tower of Hanoi problem there are $n = 2m$ rings and 4 pegs. Show that the rings can be moved from Peg 1 to Peg 4 in at most $3(2^m - 1)$ moves.

2. Show that the number of sequences out of $\{a, b, c\}^n$ which do not contain a consecutive sub-sequence of the form abc satisfies the recurrence $b_0 = 1, b_1 = 3, b_2 = 9$ and

$$b_n = 2b_{n-1} + c_n \quad (1)$$
$$c_n = c_{n-1} + b_{n-2} + c_{n-2} + b_{n-3} \quad (2)$$

   where c_n is the number of such sequences that start with a.

   Now find a recurrence only involving b_n, by using (1) to eliminate c_n from (2).

3. Let $a_0, a_1, a_2, \ldots$ be the sequence defined by the recurrence relation

$$a_n + 4a_{n-1} + 3a_{n-2} = n + 1 \quad \text{for } n \geq 2$$

   with initial conditions $a_0 = 1$ and $a_1 = 4$. Determine the generating function for this sequence, and use the generating function to determine a_n for all n.