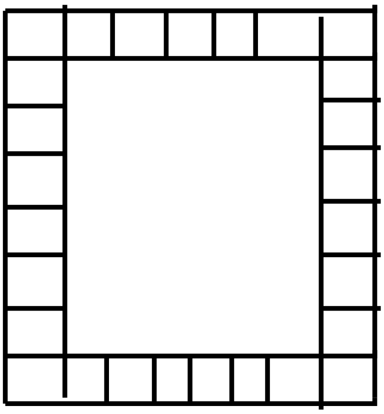
21-301 Combinatorics
Homework 11
Due: Monday, November 22



1. How many ways are there of k-coloring the squares of the above diagram if the group acting is e_0, e_1, e_2, e_3 where e_j is rotation by $2\pi j/4$. Assume that instead of 28 squares there are $4n-4$.

   Solution:

g	e_0	e_1	e_2	e_3
$\lvert Fix(g)\rvert$	k^{4n-4}	k^{n-1}	k^{2n-2}	k^{n-1}

   So the total number of colorings is
   $$\frac{k^{4n-4}+k^{n-1}+k^{2n-2}+k^{n-1}}{4}.$$

2. How many ways are there of k-coloring the squares of the same diagram if the group acting is $e_0, e_1, e_2, e_3, p, q, r, s$ where p, q, r, s are horizontal, vertical, diagonal reflections.

   Solution:

   n **even:**

g	e_0	e_1	e_2	e_3	p	q	r	s
$\lvert Fix(g)\rvert$	k^{4n-4}	k^{n-1}	k^{2n-2}	k^{n-1}	k^{2n-2}	k^{2n-2}	k^{2n-1}	k^{2n-1}

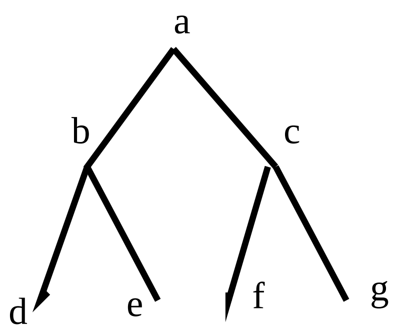
   So the total number of colorings is
   $$\frac{k^{4n-4}+k^{n-1}+k^{2n-2}+k^{n-1}+k^{2n-2}+k^{2n-2}+k^{2n-1}+k^{2n-1}}{8}.$$

   n **odd**

g	e_0	e_1	e_2	e_3	p	q	r	s
$\lvert Fix(g)\rvert$	k^{4n-4}	k^{n-1}	k^{2n-2}	k^{n-1}	k^{2n-1}	k^{2n-1}	k^{2n-1}	k^{2n-1}

   So the total number of colorings is
   $$\frac{k^{4n-4}+k^{n-1}+k^{2n-2}+k^{n-1}+k^{2n-1}+k^{2n-1}+k^{2n-1}+k^{2n-1}}{8}.$$

3. How many ways are there of k-coloring the 7 vertices of the tree below if the group acting is has elements e, g_a, g_b, g_c where e is the identity and g_x rigidly rotates the tree below x.



Solution:

g	e	g_a	g_b	g_c
$\|Fix(g)\|$	k^7	k^4	k^6	k^6

So the total number of colorings is

$$\frac{k^7 + k^4 + k^6 + k^6}{4}.$$