21-301 Combinatorics Homework 10 Due: Monday, November 15

1. Consider the following take-away game: $p \ge 3$ is a prime number. There is a pile of *n* chips. A move consists of removing p^k chips for some $k \ge 1$. Prove that the Sprague-Grundy numbers g(n) for $n \ge 0$ are given by

$$g(n) = \begin{cases} 0 & n = 0, 1, \dots, p-1 \mod 2p \\ 1 & n = p, p+1, \dots, 2p-1 \mod 2p \end{cases}.$$

- 2. Consider the following game: There is a pile of n chips. A move consists of removing any *proper* factor of n chips from the pile. (For the purposes of this question, a proper factor of n, is any factor, including 1, that is strictly less than n). The player to leave a pile with one chip wins. Determine the N and P positions and a winning strategy from an N position.
- 3. In a take-away game, the set S of the possible numbers of chips to remove is finite. Show that the Sprague-Grundy numbers satisfy $g(n) \leq |S|$ where n is the number of chips remaining.