21-301 Combinatorics Homework 1 Due: Monday, August 30

1. How many integral solutions of

 $x_1 + x_2 + x_3 + x_4 + x_5 = 100$

satisfy $x_1 \ge 3$, $x_2 \ge 10$, $x_3 \ge -3$, $x_4 \ge 4$ and $x_5 \ge 0$?

2. Show that if $n \ge q \ge 0$ then

$$\sum_{k=0}^{\ell} \binom{\ell-k}{m} \binom{q+k}{n} = \binom{\ell+q+1}{m+n+1}.$$

3. How many ways are there of placing k 1's and n - k 0's at the vertices of each of the cycles in the diagram below so that each 1 is separated by at least one 0? Thus there will either be 2k 1's altogether, when the common vertex has a 0 on it, or 2k - 1 1's altogether, when the common vertex has a 1 on it.



Each cycle has n vertices. There are 2n - 1 vertices altogether.