

8/27/10

Vandermonde's Identity

$$\sum_{r=0}^k \binom{m}{r} \binom{n}{k-r} = \binom{m+n}{k}$$

Diagram illustrating the combinatorial interpretation:

- A horizontal line segment connects two points labeled b and $m+n$.
- A vertical line segment connects point b to the horizontal line.
- The interval between b and $m+n$ is divided into two regions by a red bracket: one from b to m , and another from m to $m+n$.
- Red arrows point from the term $\binom{m}{r}$ to the interval $[b, m]$ and from the term $\binom{n}{k-r}$ to the interval $[m, m+n]$.
- A red arrow points from the right side of the equation to the term $\binom{m+n}{k}$, which is labeled "# of k-sets in $[m+n]$ ".

$$\sum_{n=0}^{\infty} (x+1)^n = \frac{1}{1-(x+1)} = \frac{1}{(1-x)(1-y)}$$

$$(x+1) \cdots \cdots \cdots (x+1)(y+1)(y+1)$$

Cooperación entre agentes
= $x^k y^m$ donde $k+m = n$
= $(x+1)(y+1)\cdots(x+1)(y+1)$

$$1 \times 1 \times y + 1 \times y \times 1 + y \times 1 \times 1 + 1 \times 1 \times 1 \\ = (y+1)(y+1)(y+1)$$

Subsets & Varieties

$$\frac{x=1}{(1+1)^n} = 2^n$$

$$\sum_{r=0}^n \binom{n}{r}$$

of subsets of $\{n\}$

$$\frac{x=1}{(-1)^n} = 0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots - \binom{n}{n}$$

even - # odd sets

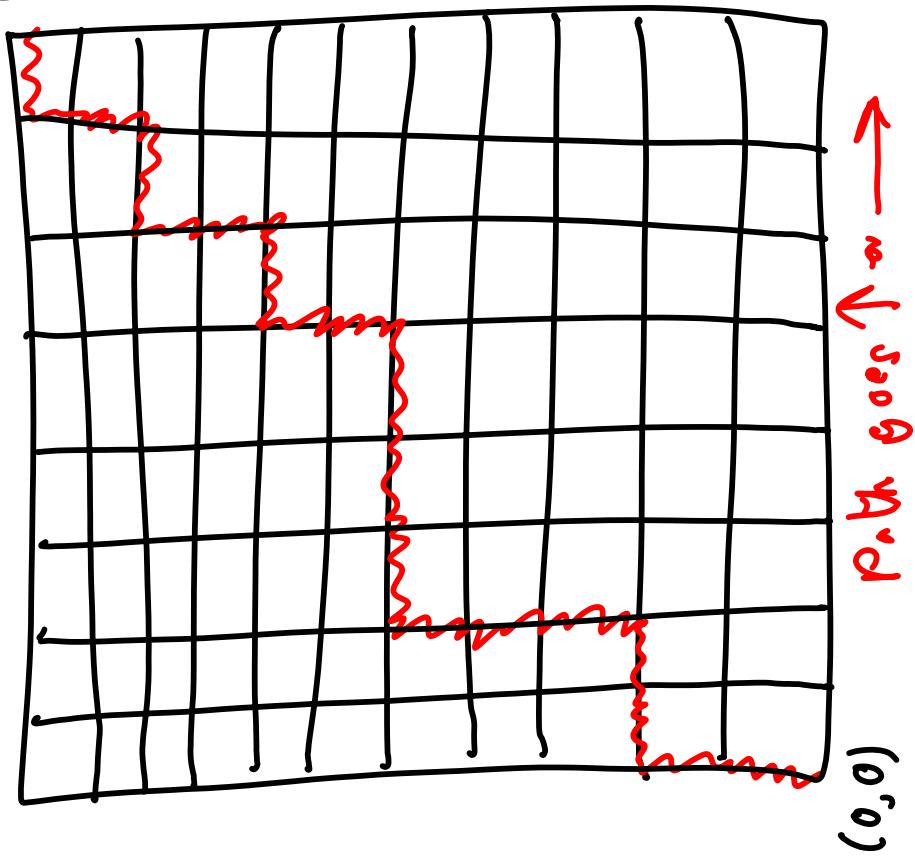
$$(1+x)^n = \sum_{r=0}^n \binom{n}{r} x^r$$

def. of exponents rule

$$n(1+x)^{n-1} = \sum_{r=0}^{n-1} r \binom{n}{r} x^{r-1}$$

$$\begin{aligned} x = -1 \\ 0 &= -\binom{n}{-1} + 2 \binom{n}{2} - 3 \binom{n}{3} + \dots \\ &= \binom{n}{-1} + 3 \binom{n}{3} + \dots = 2 \binom{n}{2} + 4 \binom{n}{4} + \dots \end{aligned}$$

(a, b)



→ $a > b$ and $b > 0$

$(0, 0)$

Paths that end

at (a, b)

① $a > b$

$\Omega_{PATHS}(a, b) =$

$\begin{cases} \{ (a, b) \} & \text{if } a = b \\ \{ (a, 0) \} & \text{if } a > b \end{cases}$

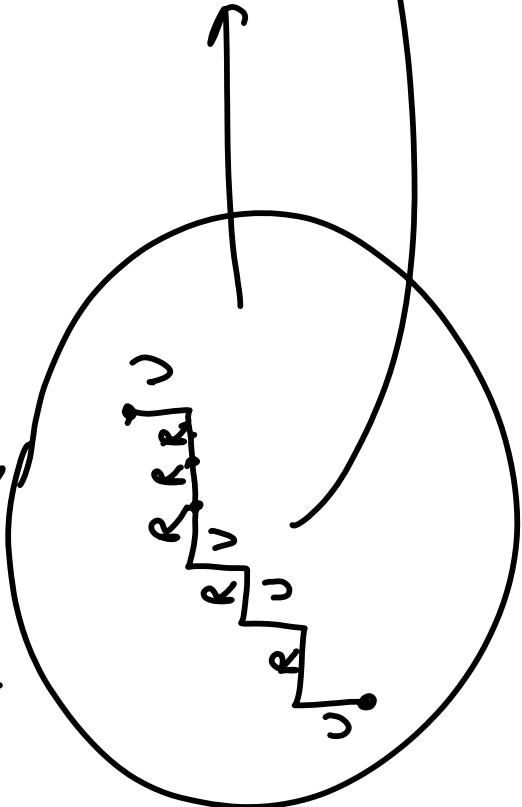
$\Omega_{PATHS}(a, b) =$

$\{ (a, 0), (1, 1), (2, 2), \dots, (a, a) \}$

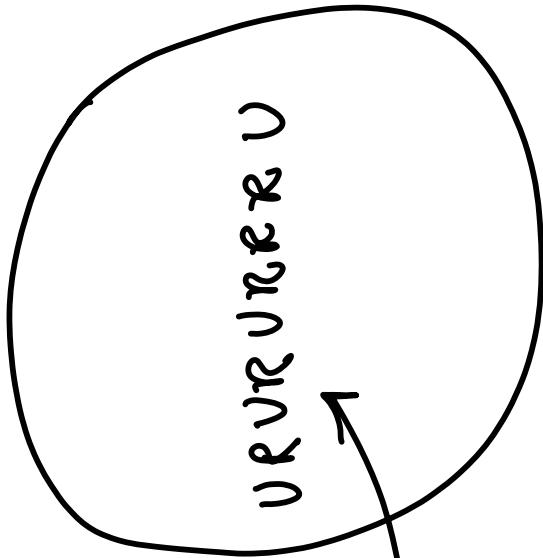
are strictly increasing

and different

(i) $\text{PAIRS}(a, b)$

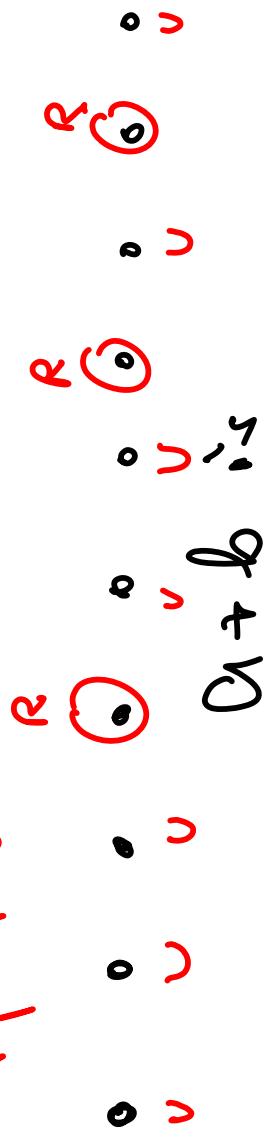


$\{\alpha, \beta\}$



paths = # strings of $a * R \in b * U$

choose a position



(2) PATHS \geq (a)

$$|\text{PATHS}^{>}(a, b)| =$$

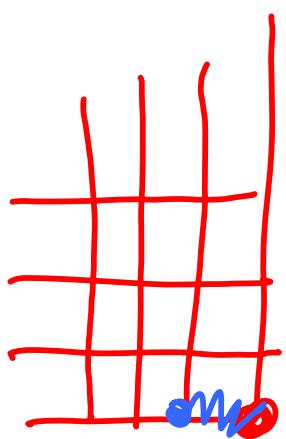
$$|\text{PATHS}((0, 1) \rightarrow (a, b))|$$

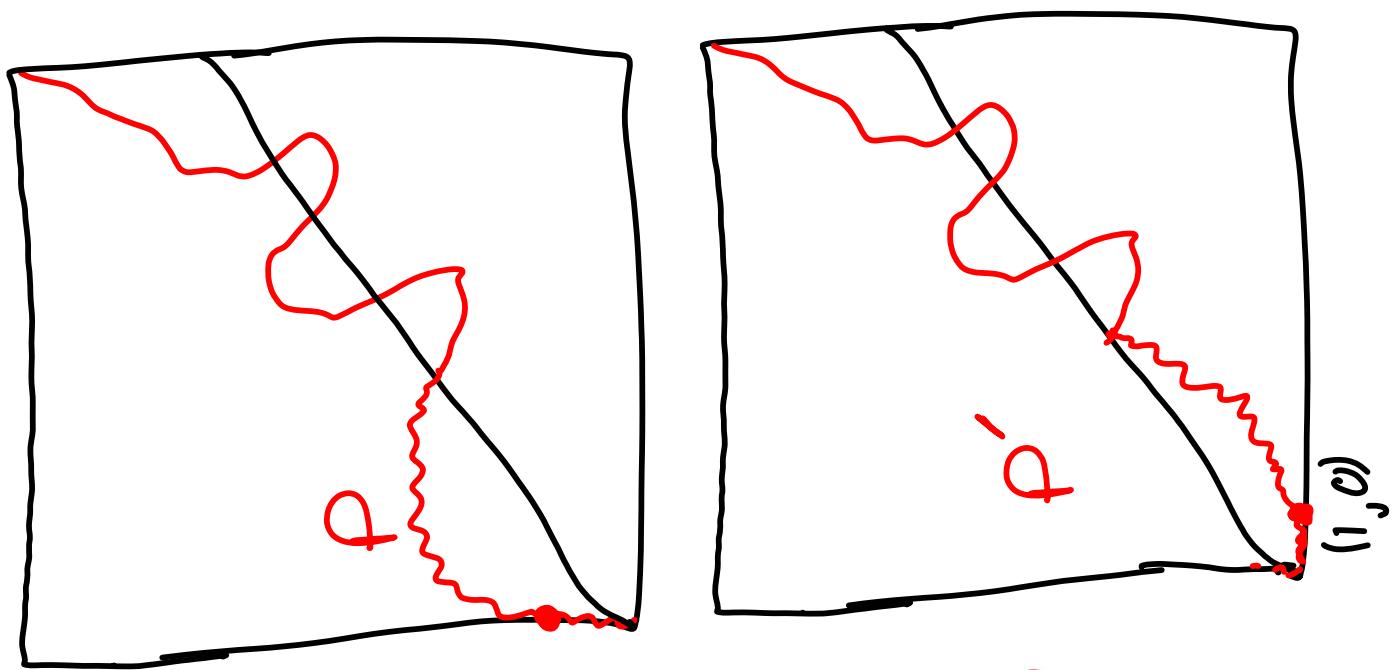
- $\text{PATHS}((0, 1) \rightarrow (a, b))$ that

cross diagonal.

1

$$\begin{aligned} & |\text{PATHS}((0, 1) \rightarrow (a, b))| \\ &= (a+b-1) \end{aligned}$$





Reflektion
verhindern

doppelte
Reflexion
verhindern

$$\begin{pmatrix} 1-a-1 \\ 1-a+b \end{pmatrix}$$

S^0

$$\frac{\log_{10} S - (\alpha, b)}{(\alpha + b - 1)} \geq \frac{(\alpha + b - 1) - (\alpha - 1)}{(\alpha + b)} = \frac{b - \alpha}{\alpha + b}$$



$$\begin{aligned}
 & \text{Numerical} \\
 & \text{Calculus} \\
 & \frac{1}{a+1} \int_0^a (2x) dx = \frac{1}{a+1} \left[x^2 \right]_0^a = \frac{a^2}{a+1} \\
 & - \int_0^a x dx = \frac{1}{2} \left[x^2 \right]_0^a = \frac{a^2}{2} \\
 & \left(a \int_0^a x dx \right) = \frac{a^3}{3} + \frac{a^2}{2} - \frac{a^2}{2} = \frac{a^3}{3}
 \end{aligned}$$

$\mu_{\text{numerical}} > \mu_{\text{calculus}}$

numerical
calculus