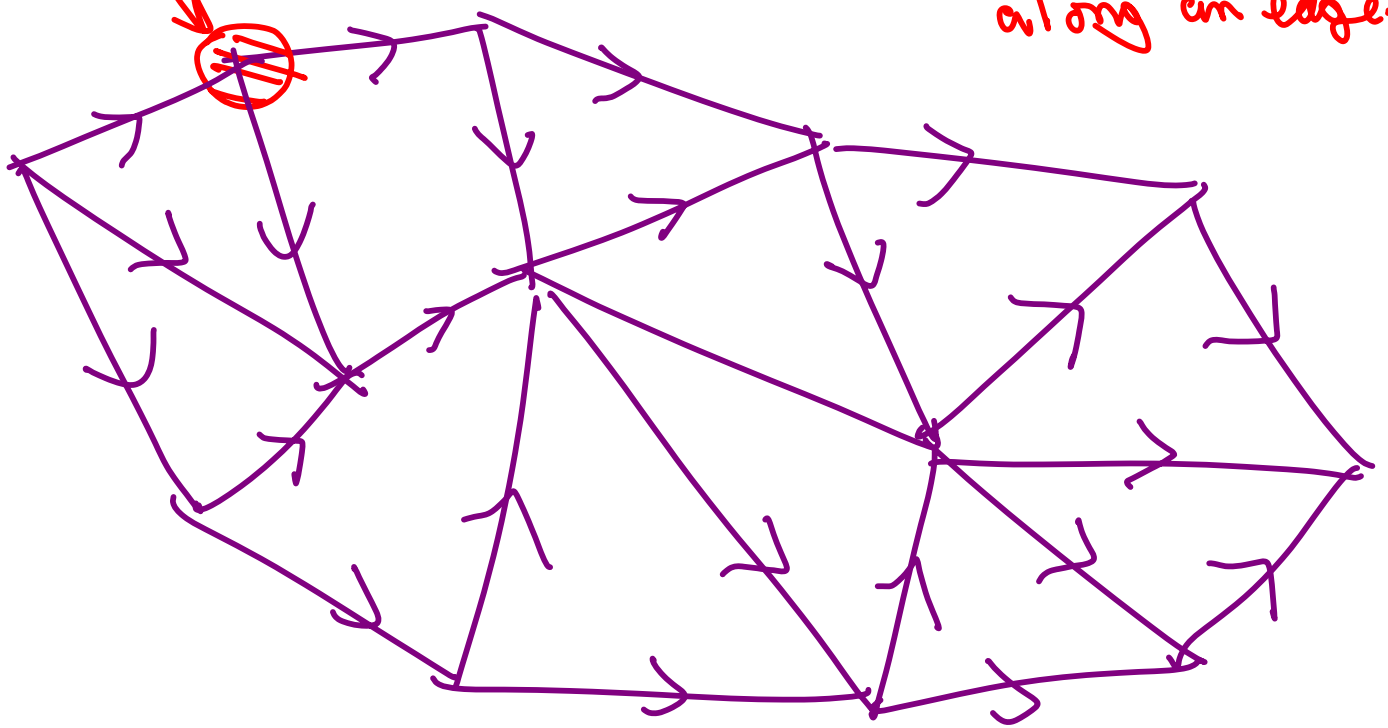
11/11/10

acyclic DAG

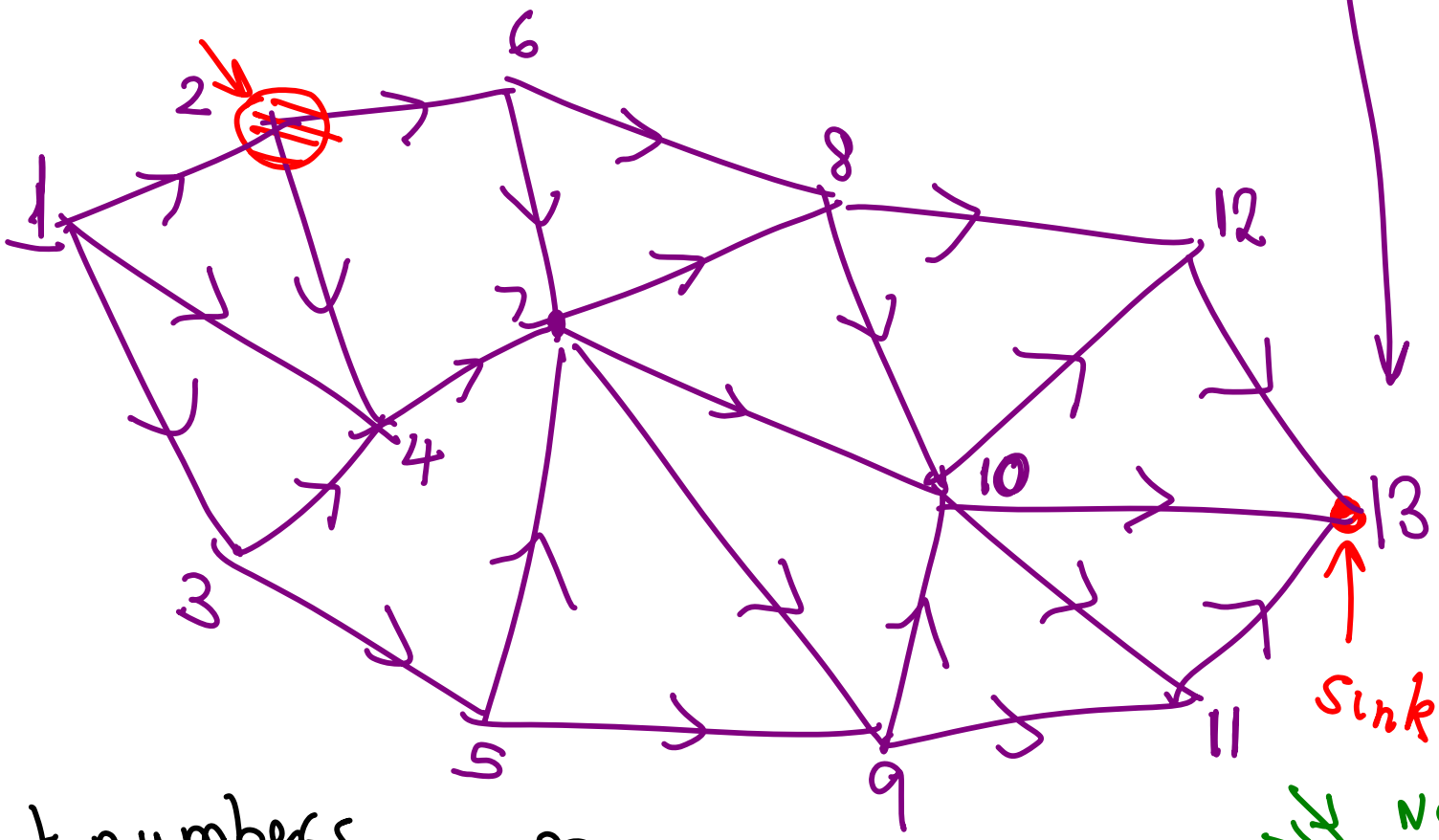
Represent a game by a digraph.

Move, move token along an edge.

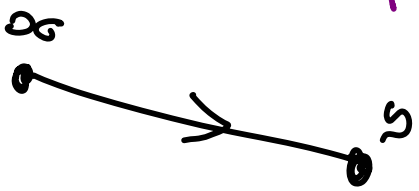


A vertex for every position.

Topological Numbering

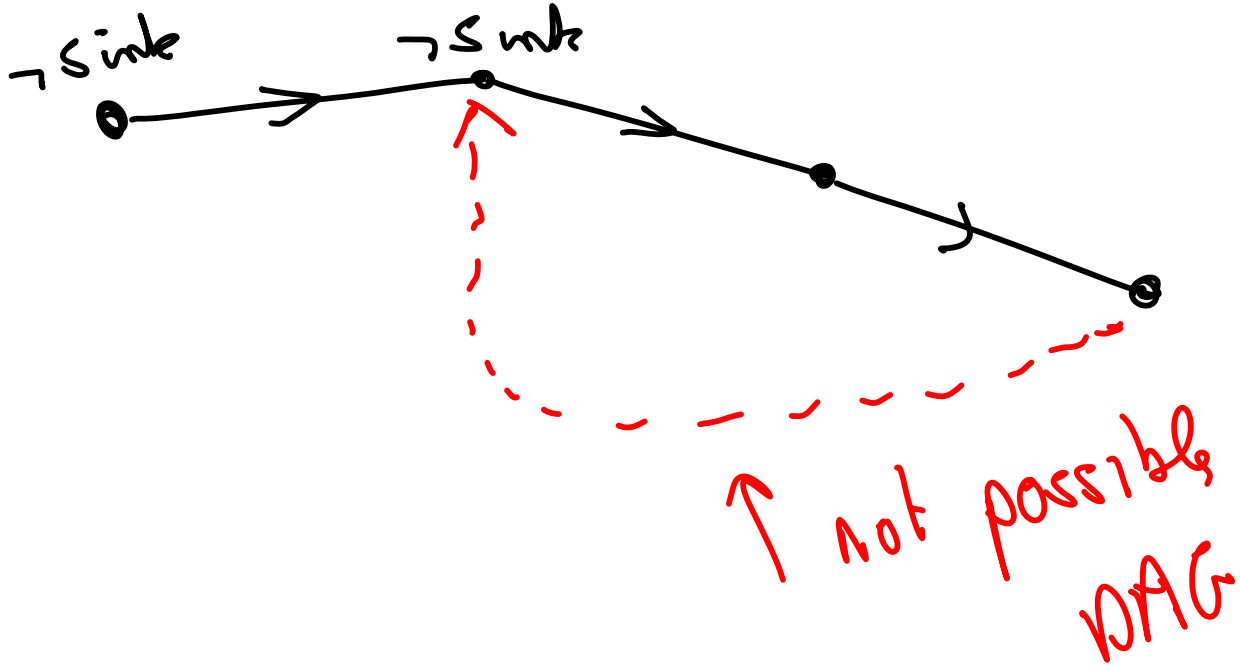


Put numbers on vertices

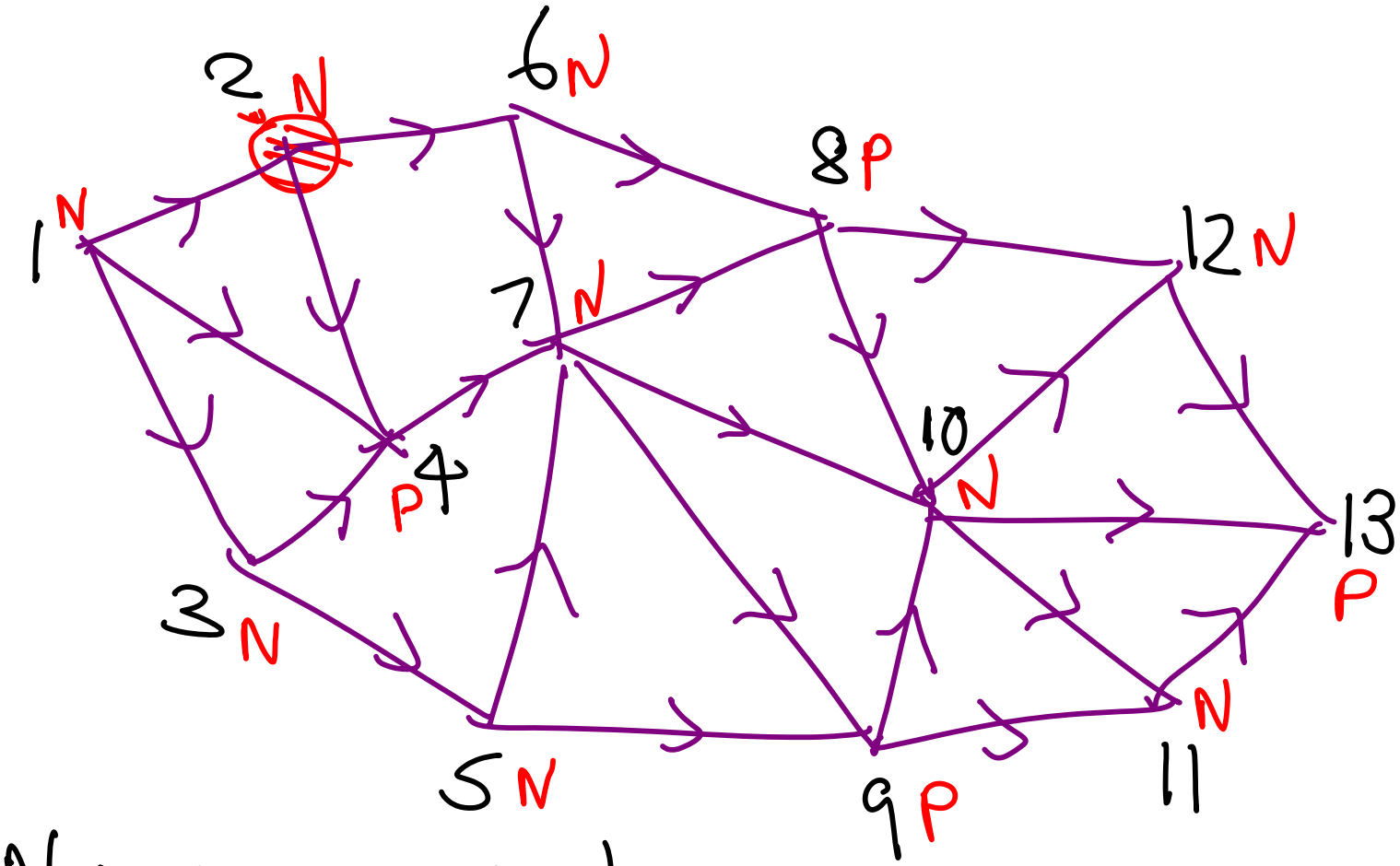


$x < y$

A DAG has at least one sink:

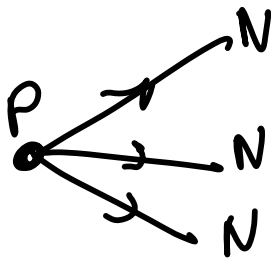


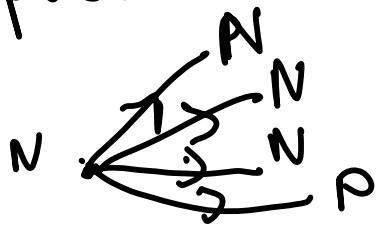
Sink $\in P$



N = winning positions

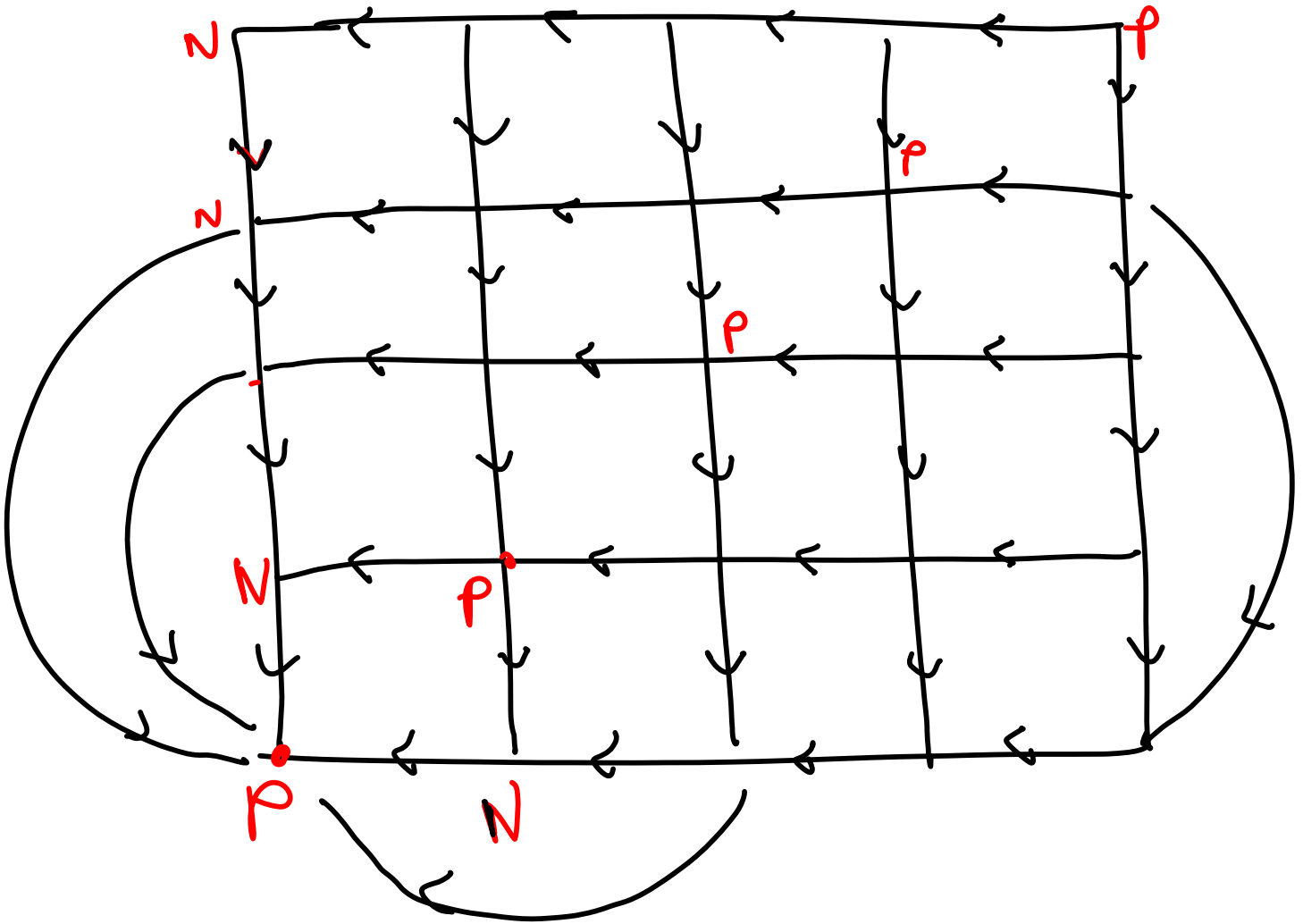
P = losing positions





Nim with 2 piles

Many edges missing from diagram.



Sums of games

$G_1, G_2, \ldots, G_p$ are games.

$$G = G_1 \oplus G_2 \oplus \cdots \oplus G_p$$

Represent by a digraph D

$$G_i \equiv D = (X_i, A_i)$$

$$G = (X_1 \times X_2 \times \cdots \times X_p, A)$$

$(x_1, x_2, \ldots x_i, \ldots x_p)$

$\downarrow$

$(x_1, x_2, \ldots x_v', \ldots x_p)$

$x_v' \in N^+(x_i)$

Choose $i = 3$ say

play a move in G_3 to remove

something from pile 3

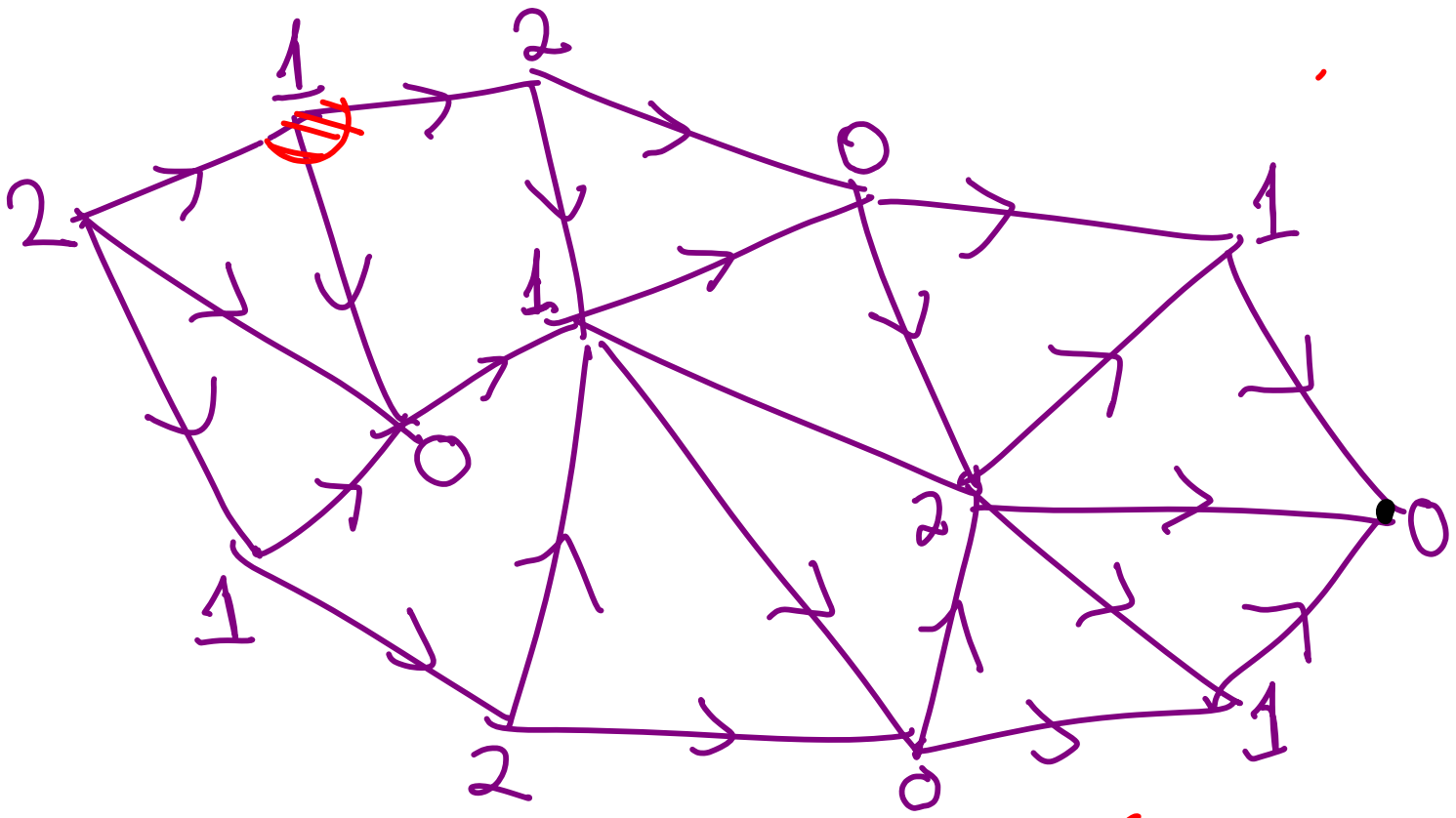
Sprague-Grundy Numbers.

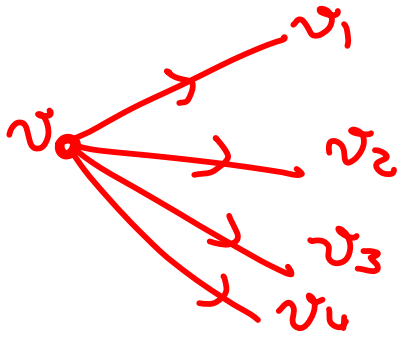
With these we can play the Sum of games.

We give each position a number

$v \in P \Leftrightarrow g(v) = 0$

SG number





$SG(v) = \max\{SG(v_1), SG(v_2), SG(v_3), SG(v_4)\}$

minimum excluded number.

$$v \in P \iff g(v) = 0$$