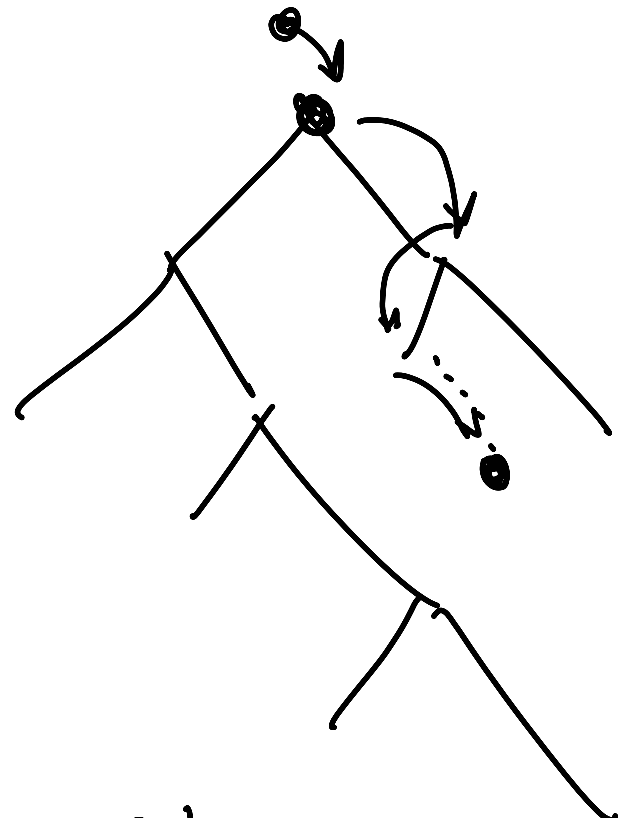
9\30\09



RLRLRRLR

Add n items.

D_n = depth of tree

Claim: Fix $t > 0$.

$$P[D_n \geq t] \leq \left(n 2^{-(t-1)/2}\right)^t$$

Proof

$$\Omega = \{L, R\}^{n^2}$$

$$\text{DEEP} = \{D_n \geq t\}$$

$P \in \{L, R\}^t$ — path in tree.

$S \subseteq [n]$, $|S| = t$ — items that produce P.

$DEEP(P, S) =$

the particles $S = \{s_1, s_2, \dots, s_t\}$

follow P in the lines.

$$DEEP = \bigcup_{P} \bigcup_{S} DEEP(P, S)$$

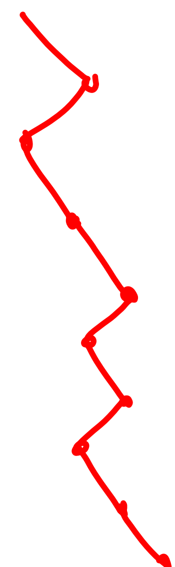
$$P(\mathrm{DEEP}) = P\Big(\bigcup_{P,S} \mathrm{DEEP}(P,S)\Big)$$

$$\leq \sum_{P} \sum_{S} P(\mathrm{DEEP}(P,S))$$

$$= \sum_{P} \sum_{S} 2^{-(1+2+\cdots+t)}$$

$$= \sum_{P} \sum_{S} 2^{-t(t+1)/2}$$

$$= 2^t \times \binom{n}{t} \times 2^{-t(t+1)/2}$$



$$= 2^t \times \binom{n}{t} \times 2^{-t(t+1)/2}$$

$$\leq 2^t \times n^t \times 2^{-t(t+1)/2}$$

$$= \left(n\, 2^{-(t-1)/2}\right)^t$$

$$\square$$

$t = A \log_2 n$

$$P(D_n \geq A \log_2 n) \leq \left(2n^{1-A/2}\right)^{A \log_2 n}$$

Conditional Probability

P

$$P_A(\omega) = \frac{P(\omega)}{P(A)} \rightarrow (A, P_A)$$

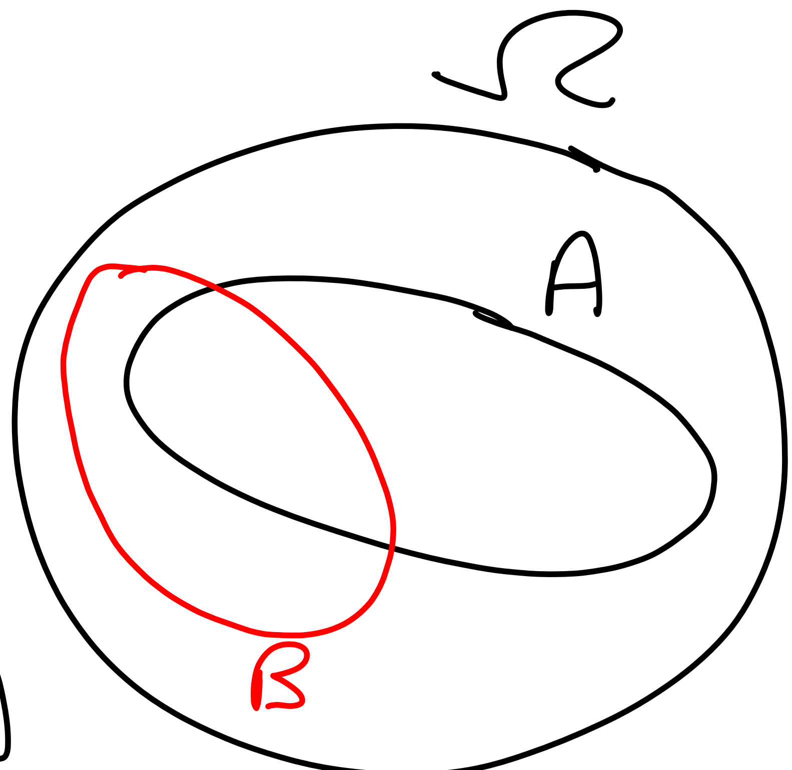
conditional probability

is a probability space.

$$P(B|A) = P_A(B)$$

Probability of B given A

"conditional probability"



$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$P_A(A\cap B)$

Family has 2 children

Each child is equally likely to be a boy or a girl.

$$\Omega = \{BB, BG, \underset{\uparrow \text{1st child}}{G}\underset{\uparrow \text{2nd child}}{B}, GG\}$$

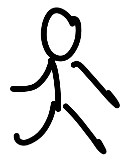
$$\Pr(BB \mid \underbrace{\text{at least one boy}}_{A}\}$$

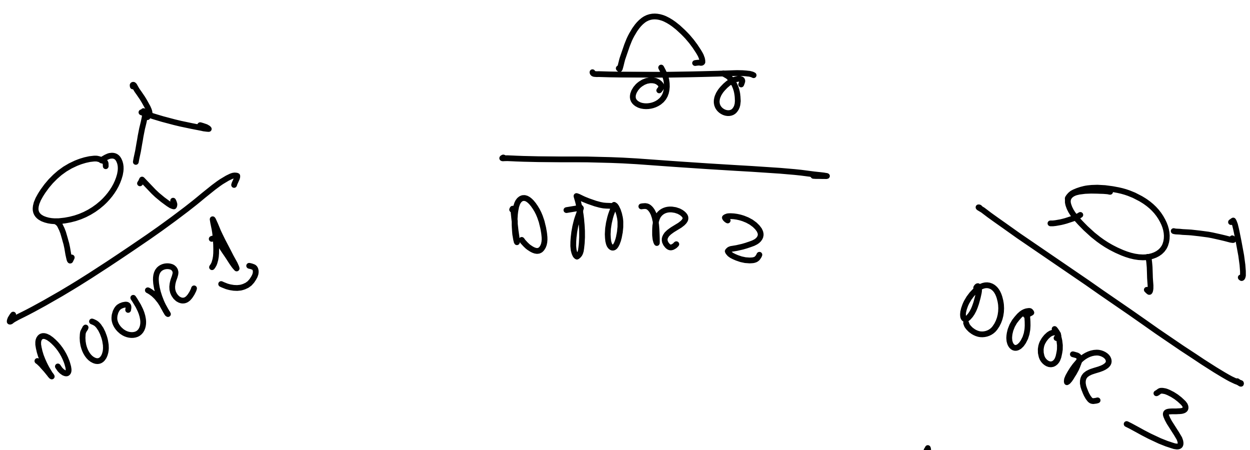
$$A = \{BB, BG, GB\}$$

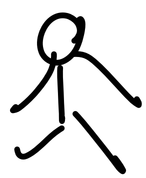
$$P(BB|A) = \frac{\Pr(BB \cap A)}{P(A)} = \frac{P(BB)}{P(A)}$$

$$= 1/3$$

Monty Hall Paradox

Host  Monty Hall $\equiv$ M



Contestant C = 

(i) C chooses a door

(ii) Monty opens a door with a goat

(iii) M says - do you want to switch?

Assume $a = \underline{1}$

$$P = \quad \tfrac{1}{3}\times\tfrac{1}{2} \quad \tfrac{1}{3}\times\tfrac{1}{2} \quad \tfrac{1}{3} \quad \tfrac{1}{3}$$

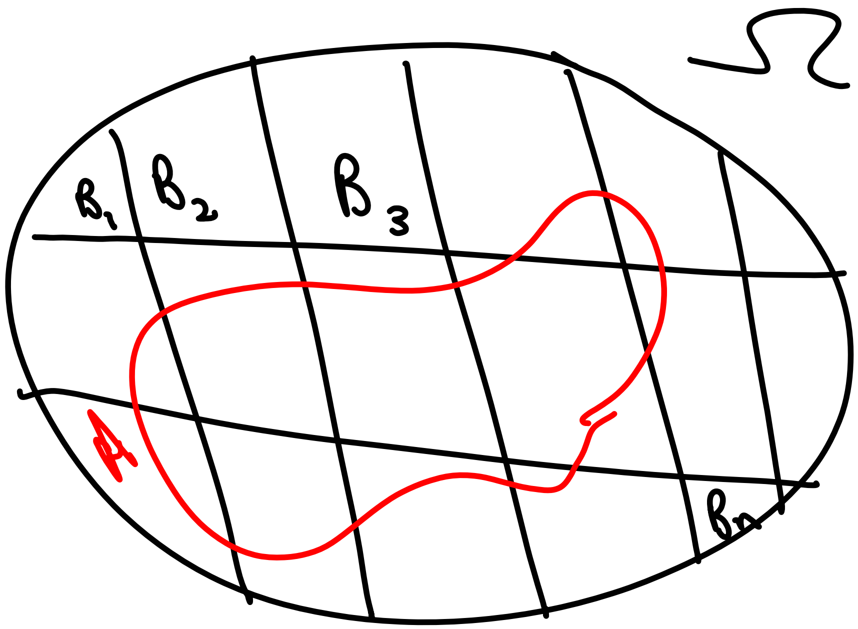
$$\Omega = \{12, 13, 23, 32\}$$

i car is behind i

j Monty opens j

$$P(1) = P(12) + P(13) = \frac{1}{3}$$

Law of total probability



$$P(A) = \sum_{i=1}^{n} P(A \mid B_i)\, P(B_i)$$

$$\sum_{i=1}^{n} P(A|B_i)\,P(B_i)$$

$$= \sum_{i=1}^{n} P(A \cap B_i)$$

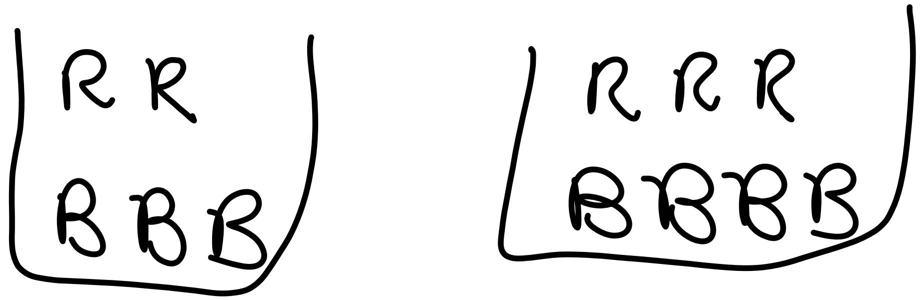
$$= P\left(A \cap \bigcup_{i=1}^{n} B_i\right)$$

$$= P(A)$$

disjoint events.

$A \cap B = \emptyset$

$P(A \cup B) = P(A) + P(B)$



b_1 chosen randomly from Urn 1
placed in Urn 2.

b_2 is chosen randomly from Urn 2.

$\Pr(b_2 \text{ is blue}) = ?$

$A = \{b_2 = B\}$

$B_1 = \{b_1 = B\}$

$B_2 = \{b_1 = R\}$

$$P(A) = \underset{5/8}{P(A|B_1)}\,\underset{3/5}{P(B_1)} + \underset{1/2}{P(A|B_2)}\,\underset{2/5}{P(B_2)}$$