9/09/09

Scrambled Allocations

n boxes

$2n$ distinguishable balls.

① ② . . . . . . ($2n$)

Put balls $\longrightarrow$ boxes

2 balls per box.

ways of putting balls in

is

$$\frac{(2n)!}{2^n}$$

Allocation is scrambled

if no box i contains

b_{2i-1}, b_{2i}

$\neg \equiv$ not

Scrambled $\equiv$

not (Box 1 contain b_1, b_2) $\neg A_1$

and

not (Box 2 contain b_3, b_4) $\neg A_2$

and

⋮

$A_i = \{$ Box i contain $b_{2i-1}, b_{2i} \}$

scrambled allocations

$$= \sum_{S \subseteq [n]} (-1)^{|S|} |A_S|$$

$A_S = \{$ allocations such that box i, $i \in S$ contain b_{2i-1}, b_{2i} $\}$

$|A_S| =$ # of allocations of $2(n-s)$ balls into $n-s$ boxes, $s = |S|$

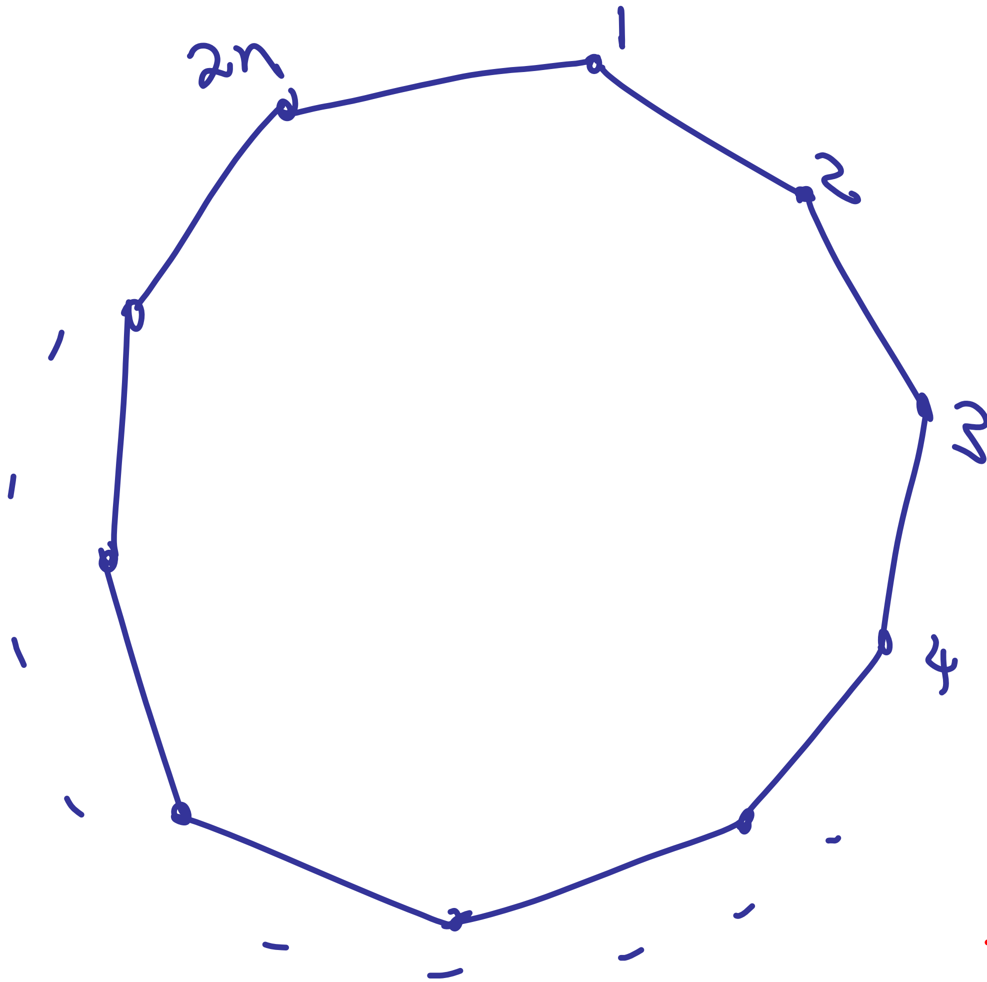
$$= \frac{(2(n-s))!}{2^{n-s}}$$

\#scrambled

$$= \sum_{S \subseteq [n]} (-1)^{|S|} \frac{(2(n-|S|))!}{2^{n-|S|}}$$

$$= \sum_{k=0}^{n} (-1)^{k} \binom{n}{k} \frac{(2(n-k))!}{2^{n-k}}$$

Problème des Ménages



n couples

(m_i, w_i)

$i = 1, \dots, n$

Problem:

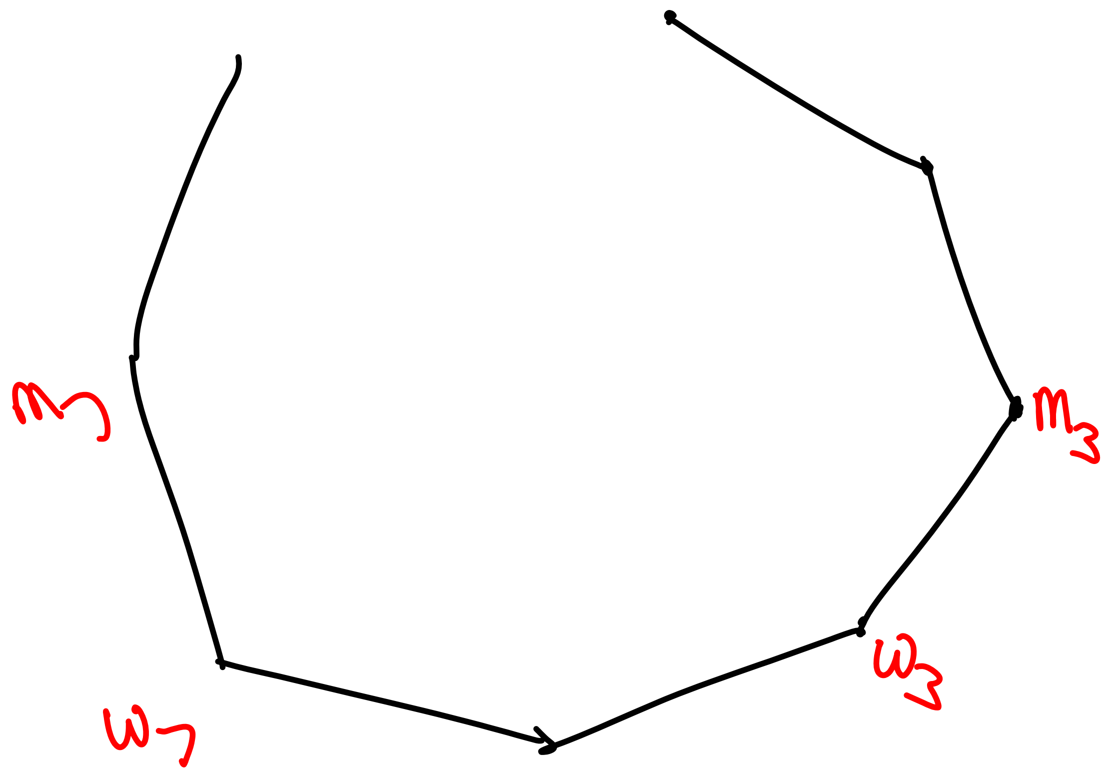
ways of seating

$m, w, m, w, \dots$

round table s.t.

that m_i does not

sit next to w_i.



forbidden

Seat arrangement is OK if

not (m_1 & w_1 next to each other)

and

not (m_2 & w_2 next to each other)

and

⋮

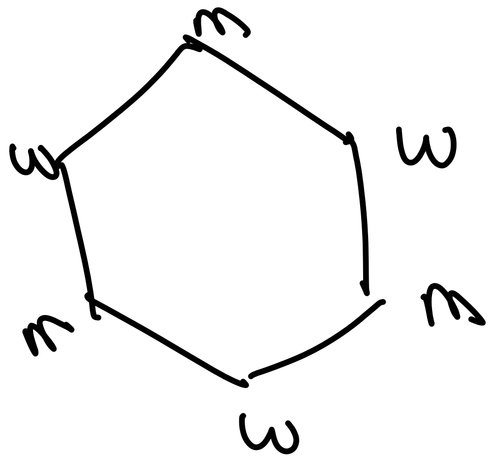
$A_i := \{$ seatings such that m_i & w_i are together$\}$

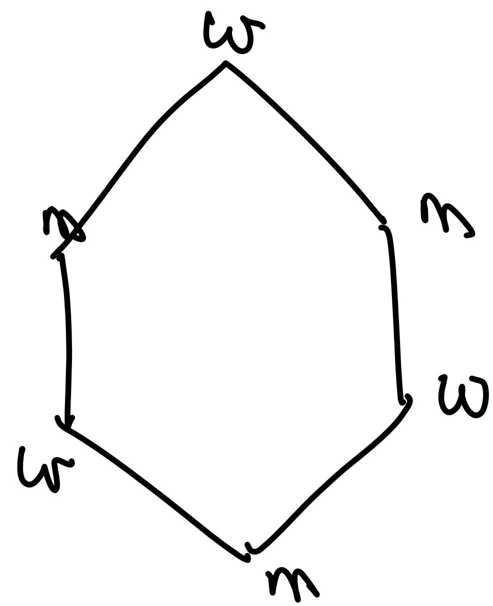
$|A_S| = ??$ $\qquad |S| = k$

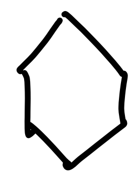
$|A_S| = 2 \times k! \times (n-k)! \times d_k$

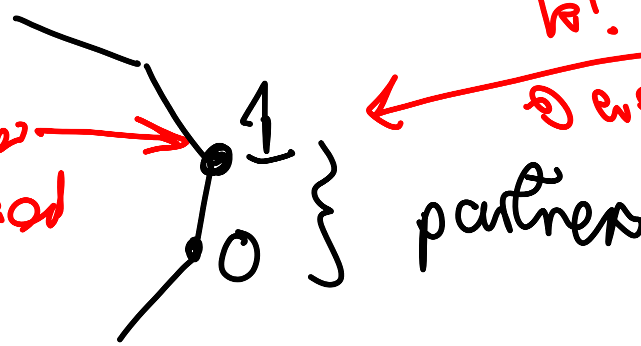
d_k = # of ways of placing k 1's on a cycle of length $2n$, no 2 1's adj.

Explanation:

2 : 

OR 

d_k : put k 1's onto 



gender is fixed

$k!$. choice of assignment

assign couple

$(n-k)!^2$ ways of assigning others

# of proper seatings is

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} \cdot 2 \cdot k! \times (n-k)!^2 \times$$

choose badly seated couple

$$d_k = \frac{2n}{2n-k}\binom{2n-k}{k}$$