10\26\09

Turan's Theorem

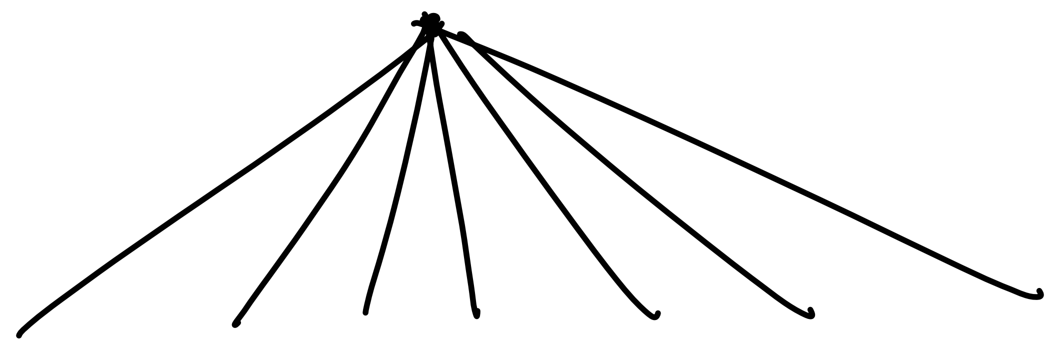
$$\alpha(G) \geq \frac{n}{\underbrace{\frac{2m}{n}} + 1}$$

size of largest independent set in G.

average degree in G.

Greedy algorithm garantees an independent set of size

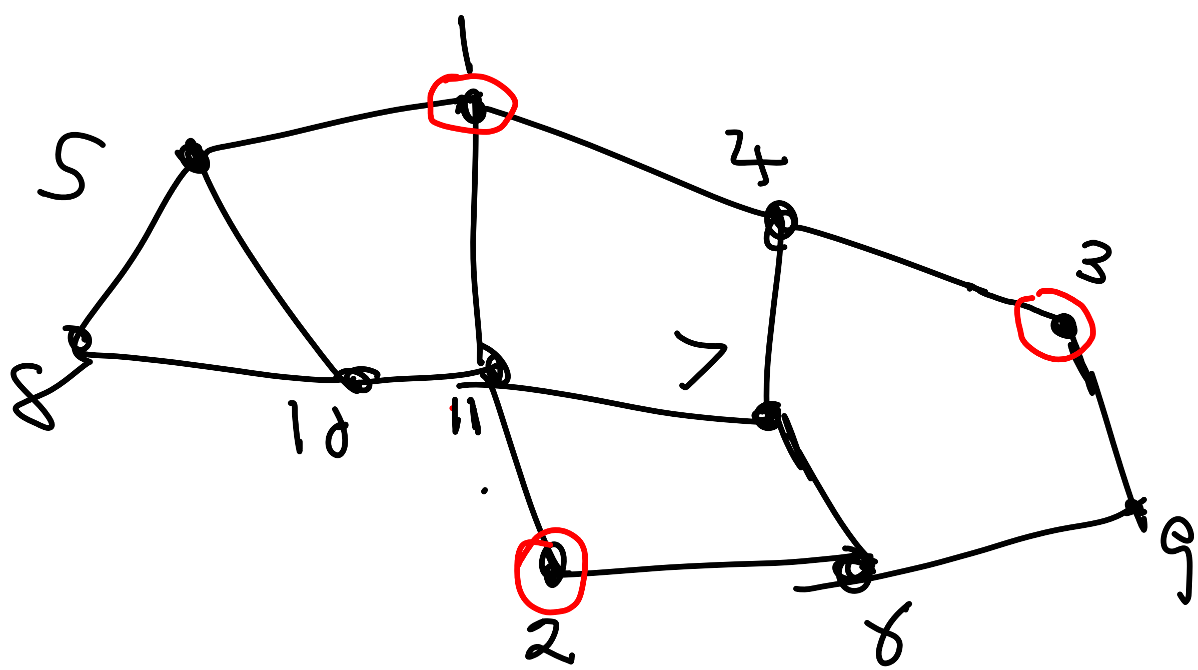
$$\frac{n}{\Delta+1}$$



Bad estimate because GREEDY is done in a bad way. Think of doing GREEDY in a random order

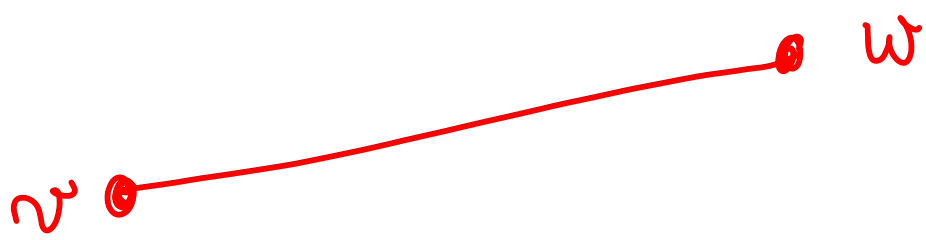
Suppose that π is a permutation on the vertex set V.

$$I(\pi) = \{ v : \pi(v) < \pi(w), \ \forall \text{ nbrs } w \text{ of } v \}$$



$I(\pi)$ is independent:

If not



$v, w \in I$

$\pi(v) < \pi(w)$

& $\pi(w) < \pi(v)$

contradiction

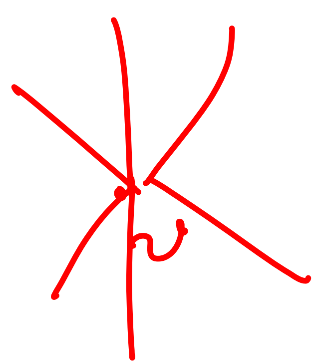
Suppose that π is a random permutation of V.

$$\delta(v) = \begin{cases} 1 & : v \in I(\pi) \\ 0 & : v \notin I(\pi) \end{cases}$$

$$|I(\pi)| = \sum_{v \in V} \delta(v)$$

$$E\left(|I(\pi)|\right) = \sum_{v \in V} E\left(\delta(v)\right)$$

$$= \sum_{v \in V} \Pr\left(\delta(v) = 1\right)$$



$$= \sum_{v \in V} \frac{1}{d(v)+1}$$

Therefore

$$\alpha(G) \geq \sum_{v \in V} \frac{1}{d(v)+1}$$

Claim: If $x_1, x_2, \dots, x_k > 0$ then

$$\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_k} \geq \frac{k^2}{x_1 + x_2 + \dots + x_k}$$

$$\sum_{v} \frac{1}{d(v)+1} \geq \frac{n^2}{2m + n}$$

Proof of claim

Claim: If $x_1, x_2, \ldots, x_k > 0$ then

$$\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_k} \geq \frac{k^2}{x_1 + x_2 + \cdots + x_k}$$

Multiply both sides by $x_1 + \cdots + x_k$

$$\sum_{1 \leq i < j \leq k} \underbrace{\left(\frac{x_i}{x_j} + \frac{x_j}{x_i} \right)}_{\geq 2} + \underset{\substack{\uparrow \\ i = j}}{k} \geq k^2$$

Finding Maximum in parallel

n numbers $x_1, \ldots, x_n$

n processors