10\21\09

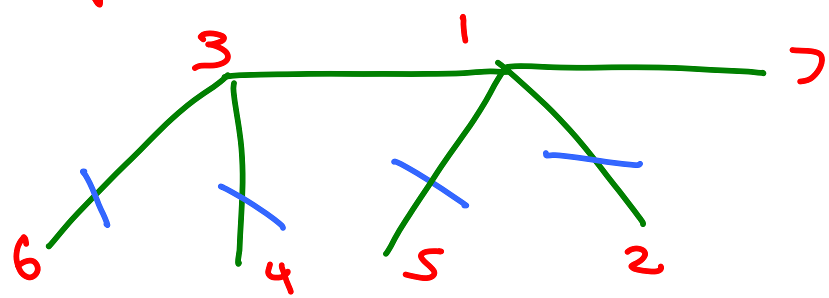
Cayley's formula for the number of distinct trees on vertex set $\{1, 2, \cdots, n\}$

$$\#\text{trees} = n^{n-2}$$

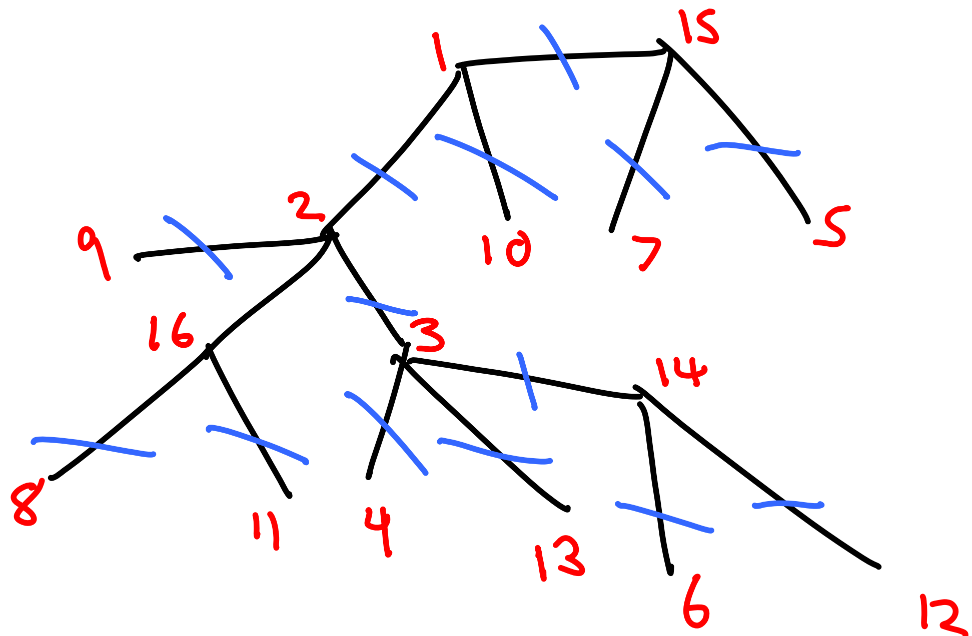
Prüfer's Correspondence.

Map: $\{\text{trees}\} \to \{[n]^{n-2}\}$

Bijection



1 3 1 3 1



3 15 14 15 16 2 1 16

14 3 3 2 1 2

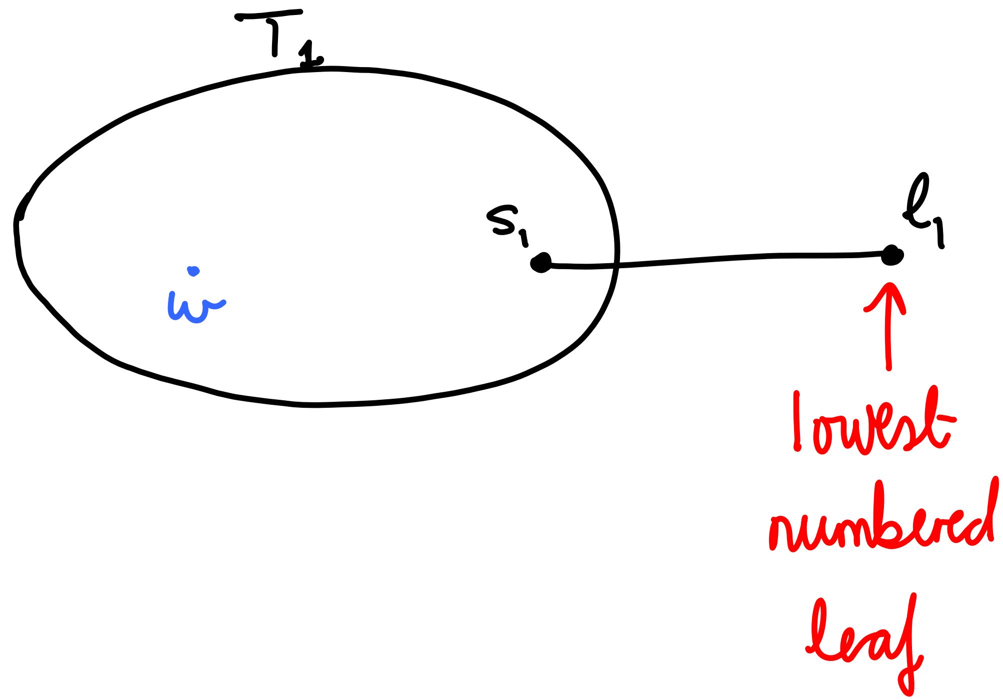
Lemma

$v \in V(T)$ appears exactly $d_T(v) - 1$ times in $\phi_v(T)$.

Proof : Induction

Base Case: $n = 2$

$$\phi_v(T) = \underbrace{\Lambda}_{\text{empty string}}$$



$$\phi_V(T) = s_1 \, \phi_{V-\ell_1}(T-\ell_1)$$

ℓ_1 appears no times ✓

s_1 appears $d_{V-\ell_1}(s_1) - 1 + 1$ times $= d_V(s_1) - \underline{1}$

Construction of ϕ_V^{-1}

<u>Proof</u>: by induction on $|V|$.

Assume that ϕ_X has an inverse if $|X| < n$.

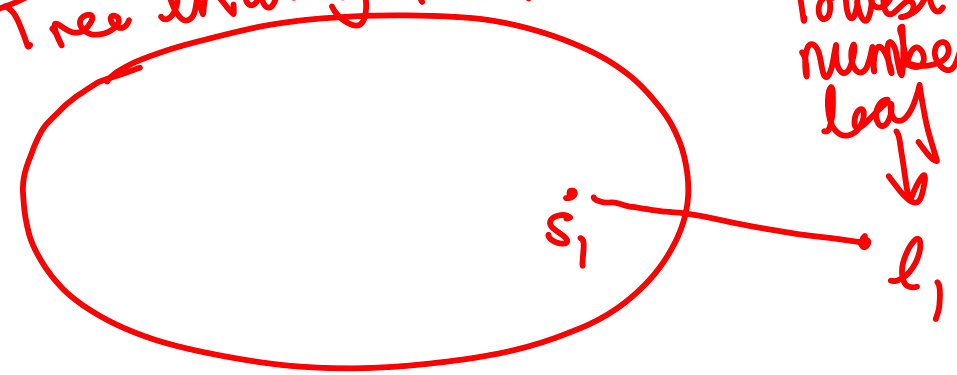
[True for $|X| \leq 2$]

$$\phi_V^{-1}(s_1 s_2 \cdots s_{n-2}) = \phi_{V-\ell_1}^{-1}(s_2 \cdots s_{n-2}) + (\ell_1, s_1)$$

lowest numbered vertex not in

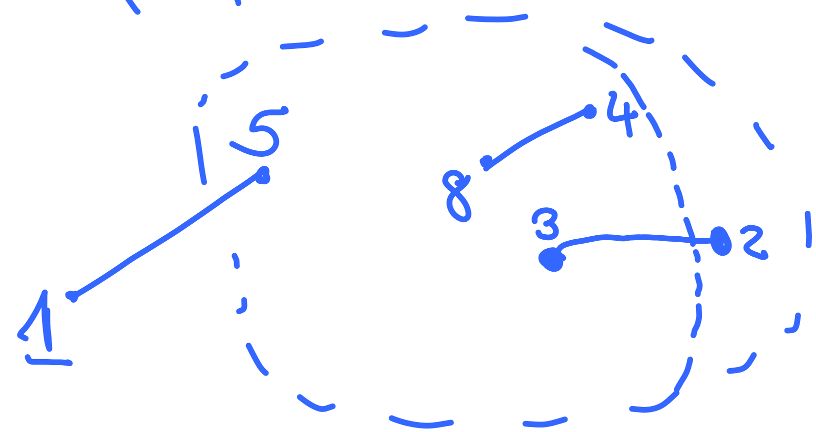
Tree that gives us

lowest numbered leaf



$\underline{n=8}$

~~5~~ ~~3~~ ~~8~~ 7 8 3



~~1~~ ~~2~~ 3 ~~4~~ 5 6 7 8