21-301 Combinatorics Homework 8 Due: Wednesday, November 12

- 1. Suppose we 2-color the edges of K_6 Red and Blue. Show that there are at least two monochromatic triangles.
- 2. Let $r_n = r(3, 3, ..., 3)$ be the minimum integer such that if we *n*-color the edges of the complete graph K_N there is a monochromatic triangle.

(a) Show that
$$r_n \le n(r_{n-1}-1)+2$$
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- (b) Using $r_2 = 6$, show that $r_n \leq \lfloor n! e \rfloor + 1$.
- 3. Let G_1, G_2 be fixed graphs. Let $r(G_1, G_2)$ be the smallest integer such that if we two-color the edges of the complete graph K_N there is a Red copy of G_1 or a Blue copy of G_2 , or both. Show that if P_3 is a path of length 3 and C_4 is a 4-cycle, then

$$r(P_3, P_3) = 5, r(P_3, C_4) = 5, r(C_4, C_4) = 6$$