21-301 Combinatorics
Homework 7
Due: Monday, November 3

1. Let $\chi(G)$ be the chromatic number of graph $G = (V, E)$. Let $\alpha(G), \kappa(G)$ denote the size of the largest independent set of G, clique of G respectively.

   Show that
$$\chi(G) \geq \max\left\{\frac{|V|}{\alpha(G)}, \kappa(G)\right\}.$$
   Show further that $\chi(G)\chi(\bar{G}) \geq |V|$. Here $\bar{G}$ is the complement of G.

2. Given any sequence of n integers, positive or negative, not necessarily all different, show that some consecutive subsequence has the property that the sum of the members of the subsequence is a multiple of n.

3. Suppose that $a_1, a_2, \ldots, a_n \in [n]$ and $b_1, b_2, \ldots, b_n \in [n]$. An interval I is a set of the form $\{i, i+1, \ldots, j\}$. Let $a_I = a_i + a_{i+1} + \cdots + a_j$ and $b_I = b_i + b_{i+1} + \cdots + b_j$. Show that there exist intervals I, J such that $a_I = b_J$.