21-301 Combinatorics Homework 6 Due: Monday, October 27

- 1. Using Cayley's formula, show that the graph obtained from  $K_n$  by deleting one edge has exactly  $(n-2)n^{n-3}$  spanning trees.
- 2. Let G = (V, E) be an *r*-regular graph with *n* vertices i.e. every vertex has degree *r*.  $S \subseteq V$  is a *dominating set* if  $w \notin S$  implies that there exists  $v \in S$  for which  $\{v, w\} \in E$ . Show, by the probabilistic method, that *G* has a dominating set of size at most  $\frac{1+\ln r}{r}n$ .
- 3. Let G = (V, E) be a graph with minimum degree at least three. Show that it contains a cycle of even length. (Hint: Consider a longest path).