21-301 Combinatorics
Homework 5
Due: Monday, October 13

1. A box has m drawers; Drawer i contains g_i gold coins, s_i silver coins and ℓ_i lead coins, for $i = 1, 2, \ldots, m$. Assume that one drawer is selected randomly and that two randomly selected coins from that drawer turn out to have the same color. What is the probability that the chosen drawer is drawer 1?

2. Let $m = \lfloor (8/7)^{n/3} \rfloor$. Show that there exist distinct sets $A_1, A_2, \ldots, A_m \subseteq [n]$ such that for all distinct $i, j, k \in [m]$ we have $A_i \cap A_j \not\subseteq A_k$.

3. A particle sits at the left hand end of a line $0 - 1 - 2 - \cdots - L$. When at 0 it moves to 1. When at $i \in [1, L-1]$ it makes a move to $i-1$ with probability $1/4$ and a move to $i+1$ with probability $3/4$. When at L it stops.

   Let E_k denote the expected number of visits to 0 if we started the walk at k.

   (a) Explain why

   $$\begin{aligned} E_L &= 0 \\ E_0 &= 1 + E_1 \\ E_k &= \frac{1}{4}E_{k-1} + \frac{3}{4}E_{k+1} \qquad for\ 0 < k < L. \end{aligned}$$

   (b) Given that $E_k = \frac{A}{3^k} + B$ is a solution to your equations for some A, B, determine A, B and hence find E_0.